Lecture (1)

Two-Port Networks

1) Introduction

- A ***port*** *is a pair of terminals through which a current may enter or leave a network.*
- *A* ***one-port network*** *is a two-terminal networks or devices or elements (such as resistors, capacitors, and inductors). Most of the circuits we have dealt with so far are two-terminal or one-port circuits as in Fig.1.1 (a).*
- *A* ***Two-port network*** *is a four terminals networks or devices or elements(as shown in Fig.1.1 (b)), such as op amps, transistors, and transformers.*
- A two-port network is an electrical network with two separate ports for input and output.
- *A* ***multi-ports network*** *is an n-ports (n-pairs of terminals) networks or devices or elements.*

Note: the current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

Fig.1.1 (a) One-port network, (b) two-port network.

Importance of two-port networks

1) Such networks are useful in communications, control systems, power systems, and electronics. For example, they are used in electronics to model transistors and to facilitate cascaded design.
2) Knowing the parameters of a two-port network enables us to treat it as a "black box" when embedded within a larger network.

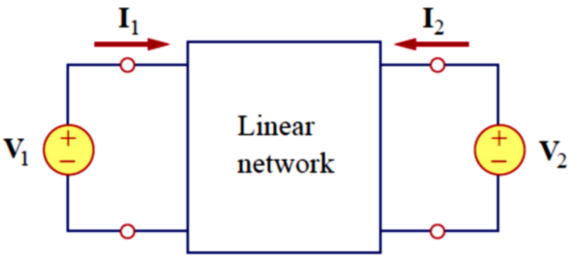
To characterize a two-port network requires that we relate the terminal quantities $\mathbf{V_1}$, $\mathbf{V_2}$, $\mathbf{I_1}$, and $\mathbf{I_2}$ in Fig.1.1 (b), out of which two are independent. The various terms that relate these voltages and currents are called ***parameters***. There are ***six sets*** of these parameters:

1) Impedance Parameters (z-parameters)
2) Admittance Parameters (y-parameters)
3) Hybrid Parameters (h-parameters)
4) Inverse Hybrid Parameters (g-parameters)
5) Transmission Parameters (ABCD or T parameters)
6) Inverse Transmission Parameters (abcd or t parameters)

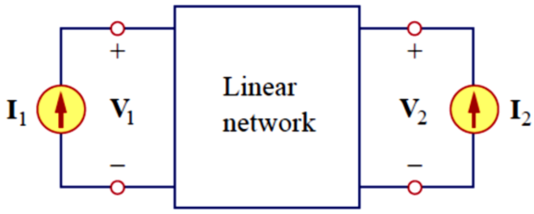
1) *Impedance Parameters (z-parameters)*

Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks.

A two-port network may be voltage-driven as in Fig.2.1 (a) or current-driven as in Fig.2.1 (b). From either Fig.2.1 (a) or (b), the terminal voltages can be related to the terminal currents as



(a)



(b)

Fig.2.1

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{aligned} \qquad (2.1)$$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \qquad (2.2)$$

where the **z** terms are called *impedance parameters*, or ***z*** *parameters*, and have units of ohms. The values of the parameters can be evaluated by setting $\mathbf{I}_1 = 0$ (input port open-circuited) or $\mathbf{I}_2 = 0$ (output port open-circuited) as shown in Fig.2.2. Thus,

$$\begin{aligned} \mathbf{z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, & \mathbf{z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{z}_{21} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0}, & \mathbf{z}_{22} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{aligned} \qquad (2.3)$$

Since the **z** parameters are obtained by open-circuiting the input or output port, they are also called the *open-circuit impedance parameters*. Specifically,

$$\begin{aligned} \mathbf{z}_{11} &= \text{Open-circuit input impedance} \\ \mathbf{z}_{12} &= \text{Open-circuit transfer impedance from port 1 to port 2} \\ \mathbf{z}_{21} &= \text{Open-circuit transfer impedance from port 2 to port 1} \\ \mathbf{z}_{22} &= \text{Open-circuit output impedance} \end{aligned} \qquad (2.4)$$

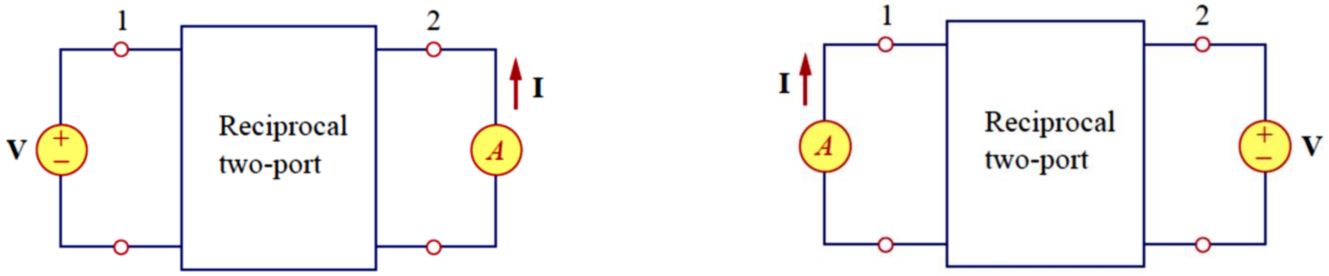
Sometimes $\mathbf{z}_{11}$ and $\mathbf{z}_{22}$ are called *driving-point impedances*, while $\mathbf{z}_{12}$ and $\mathbf{z}_{21}$ are called *transfer impedances*. When $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be *symmetrical*.

(a) (b)

Fig.2.2. Determination of the *z* parameters

1.1) Reciprocal networks

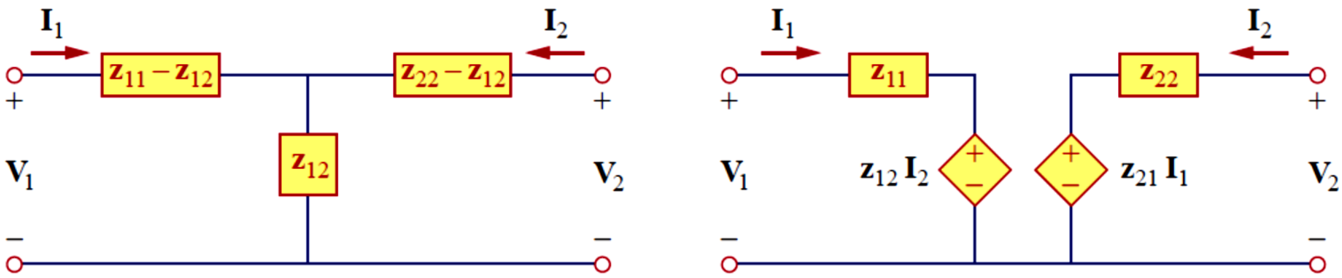
When the two-port network is linear and has no dependent sources, the transfer impedances are equal ($z_{12} = z_{21}$), and the two-port is said to be ***reciprocal***. This means that if the points of excitation and response are interchanged, the transfer impedances remain the same. As illustrated in Fig.2.3, a two-port is reciprocal if interchanging an ideal voltage source at one port with an ideal ammeter at the other port gives the same ammeter reading. The reciprocal network yields $\mathbf{V} = \mathbf{z_{12}I}$ when connected as in Fig.2.3 (a), but yields $\mathbf{V} = \mathbf{z_{21}I}$ when connected as in Fig.2.3 (b). This is possible only if $\mathbf{z_{12}} = \mathbf{z_{21}}$.



(a) (b)

Fig.2.3

Any two-port that is made entirely of resistors, capacitors, and inductors must be reciprocal. A reciprocal network can be replaced by the T-equivalent circuit in Fig. Fig.2.4(a). If the network is not reciprocal, a more general equivalent network is shown in Fig.2.4(b); notice that this figure follows directly from Eq. (2.1).



(a) (b)

Fig.2.4 (a) T-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

It should be mentioned that for some two-port networks, the ***z*** parameters do not exist because they cannot be described by Eq. (2.1). As an example, consider the ideal transformer of Fig.2.5. The defining equations for the two-port network are:

$$\mathbf{V}_1 = \frac{1}{n}\mathbf{V}_2, \qquad \mathbf{I}_1 = -n\mathbf{I}_2 \qquad \textbf{(2.5)}$$

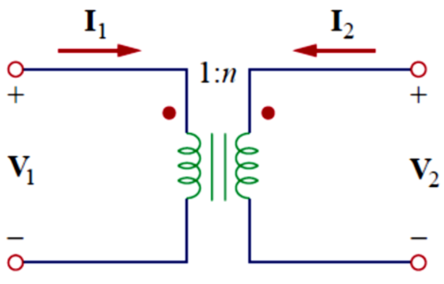
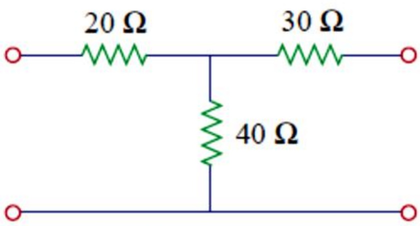


Fig.2.5

Observe that it is impossible to express the voltages in terms of the currents, and vice versa, as Eq. (2.1) requires. Thus, the ideal transformer has ***no z parameters***. However, it does have ***hybrid parameters***.

Example 1: Determine the *z* parameters for the circuit shown.

Solution:



. **Fig. 1**

■ **METHOD 1** To determine $\mathbf{z}_{11}$ and $\mathbf{z}_{21}$, we apply a voltage source to $\mathbf{V}_1$.(the input port and leave the output port open as in Fig. 2(a),Then

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{(20 + 40)\mathbf{I}_1}{\mathbf{I}_1} = 60\ \Omega$$

that is, $\mathbf{z}_{11}$ is the input impedance at port 1.

$$\mathbf{z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1} = \frac{40\mathbf{I}_1}{\mathbf{I}_1} = 40\ \Omega$$

To find $\mathbf{z}_{12}$ and $\mathbf{z}_{22}$, we apply a voltage source $\mathbf{V}_2$ to the output port ,and leave the input port open as in Fig. 2(b). Then

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{40\mathbf{I}_2}{\mathbf{I}_2} = 40\ \Omega, \qquad \mathbf{z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{(30 + 40)\mathbf{I}_2}{\mathbf{I}_2} = 70\ \Omega$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 60\ \Omega & 40\ \Omega \\ 40\ \Omega & 70\ \Omega \end{bmatrix}$$

■ **METHOD 2** Alternatively, since there is no dependent source in the given circuit, $\mathbf{z}_{12} = \mathbf{z}_{21}$ and we can use Fig.2.4(a). Comparing Fig. 1 with Fig. 2.4(a), we get

$$\mathbf{z}_{12} = 40\ \Omega = \mathbf{z}_{21}$$

$$\mathbf{z}_{11} - \mathbf{z}_{12} = 20 \quad \Rightarrow \quad \mathbf{z}_{11} = 20 + \mathbf{z}_{12} = 60\ \Omega$$

$$\mathbf{z}_{22} - \mathbf{z}_{12} = 30 \quad \Rightarrow \quad \mathbf{z}_{22} = 30 + \mathbf{z}_{12} = 70\ \Omega$$

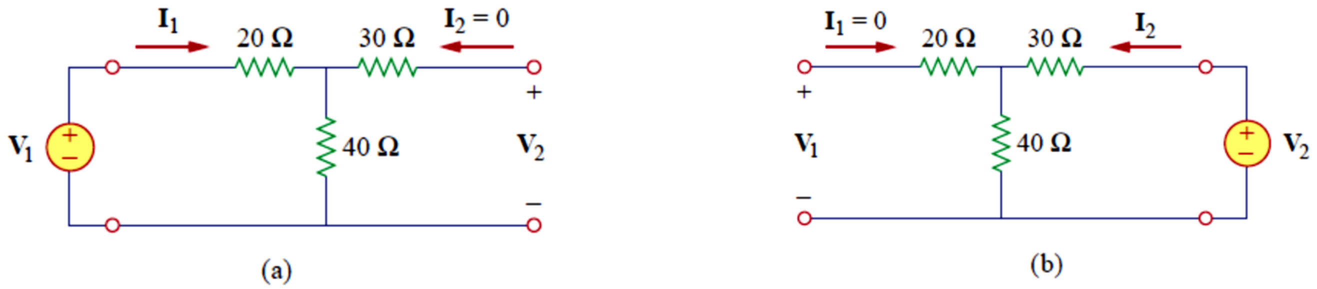
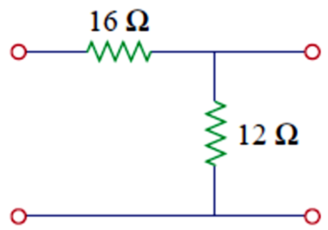


Fig.2 (a) finding z_{11} and z_{21} (b) finding z_{12} and z_{22}

H.W.1: Find the z parameters of the two-port network in Fig.

Answer:

$$z_{11} = 28\ \Omega,\ z_{12} = z_{21} = z_{22} = 12\ \Omega.$$



Example 2: Find $\mathbf{I}_1$ and $\mathbf{I}_2$ in the circuit in Fig

Solution:

This is not a reciprocal network. We may use the equivalent circuit in Fig. 2.4(b) but we can also use Eq. (2.1) directly. Substituting the given z parameters into Eq. (2.1),

$$\mathbf{V}_1 = 40\mathbf{I}_1 + j20\mathbf{I}_2 \qquad \textbf{(1)}$$

$$\mathbf{V}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \qquad \textbf{(2)}$$

Since we are looking for $\mathbf{I}_1$ and $\mathbf{I}_2$, we substitute

$$\mathbf{V}_1 = 100\underline{/0^\circ}, \qquad \mathbf{V}_2 = -10\mathbf{I}_2$$

into Eqs. (1) and (2), which become

$$100 = 40\mathbf{I}_1 + j20\mathbf{I}_2 \qquad \textbf{(3)}$$

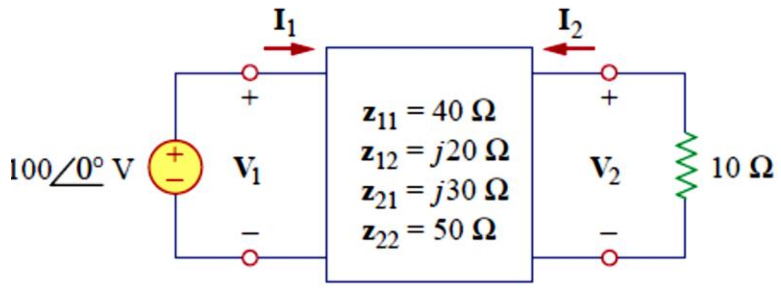
$$-10\mathbf{I}_2 = j30\mathbf{I}_1 + 50\mathbf{I}_2 \quad \Rightarrow \quad \mathbf{I}_1 = j2\mathbf{I}_2 \qquad \textbf{(4)}$$

Substituting Eq. (4) into Eq. (3) gives

$$100 = j80\mathbf{I}_2 + j20\mathbf{I}_2 \quad \Rightarrow \quad \mathbf{I}_2 = \frac{100}{j100} = -j$$

From Eq. (19.2.4), $\mathbf{I}_1 = j2(-j) = 2$. Thus,

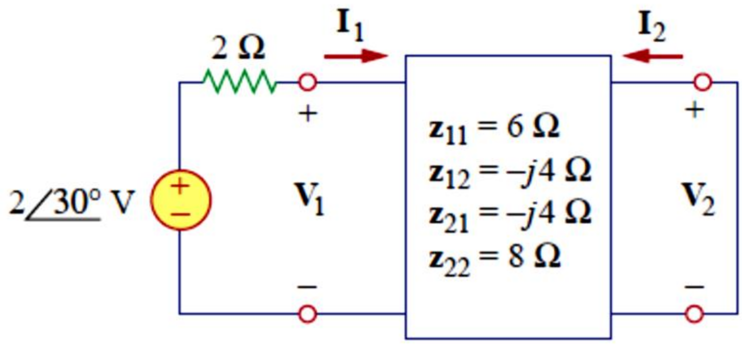
$$\mathbf{I}_1 = 2\underline{/0^\circ}\ \text{A}, \qquad \mathbf{I}_2 = 1\underline{/-90^\circ}\ \text{A}$$



H.W.2: Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$ in the two-port of Fig.

Answer:

$200\angle 30^\circ$ mA, $100\angle 120^\circ$ mA.



2) Admittance Parameters (y-parameters)

In the previous section we saw that impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need may be met by the second set of parameters, which we obtain by expressing the terminal currents in terms of the terminal voltages. In either Fig.3.1 (a) or (b), the terminal currents can be expressed in terms of the terminal voltages as

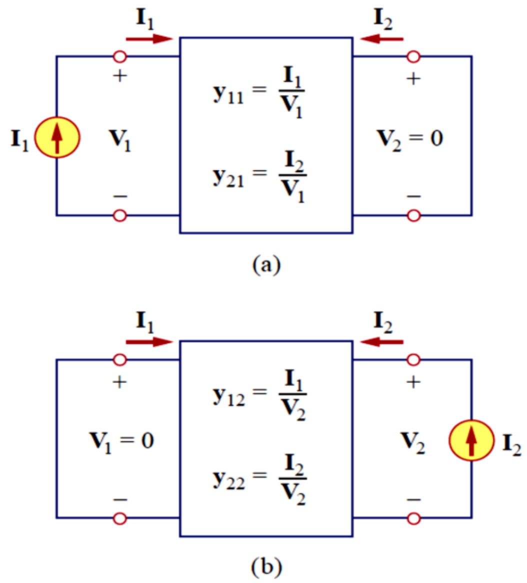


Fig.3.1 Determination of the y parameters: (a) finding y_{11} and y_{21}(b) finding y_{12} and y_{22}

$$\boxed{\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned}} \qquad (3.1)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad (3.2)$$

The terms are known as the ***admittance parameters*** (or, simply, ***y parameters***) and have units of siemens.

The values of the parameters can be determined by setting $V_1 = 0$ (input port short-circuited) or $V_2 = 0$ (output port short-circuited). Thus,

$$\boxed{\begin{aligned} y_{11} &= \left.\frac{I_1}{V_1}\right|_{V_2=0}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \\ y_{21} &= \left.\frac{I_2}{V_1}\right|_{V_2=0}, \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \end{aligned}} \qquad (3.3)$$

Since the y parameters are obtained by short-circuiting the input or output port, they are also called the ***short-circuit admittance parameters***. Specifically,

$$\begin{aligned} y_{11} &= \text{Short-circuit input admittance} \\ y_{12} &= \text{Short-circuit transfer admittance from port 2 to port 1} \\ y_{21} &= \text{Short-circuit transfer admittance from port 1 to port 2} \\ y_{22} &= \text{Short-circuit output admittance} \end{aligned} \qquad (3.4)$$

For a linear two-port network and has no dependent sources, the transfer admittances are equal ($y_{12} = y_{21}$). This can be proved in the same way as for the z parameters. A reciprocal network ($y_{12} = y_{21}$) can be modeled by the Π-equivalent circuit in Fig.3.2 (a). If the network is not reciprocal, a more general equivalent network is shown in Fig.3.2 (b).

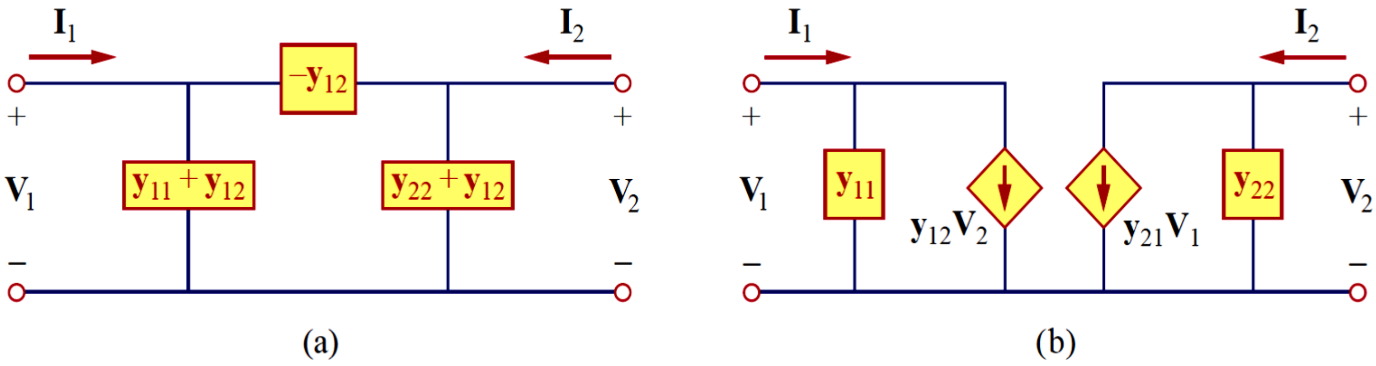


Fig.3.2 (a) Π-equivalent circuit (for reciprocal case only), (b) general equivalent circuit.

Example 3: Obtain the y parameters for the network shown in Fig.

Solution:

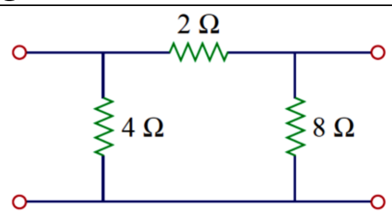


Fig.1

■ **METHOD 1** To find $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as in Fig. 2(a). Since the 8-Ω resistor is short-circuited, the 2-Ω resistor is in parallel with the 4-Ω resistor. Hence,

$$\mathbf{V}_1 = \mathbf{I}_1(4 \parallel 2) = \frac{4}{3}\mathbf{I}_1, \qquad \mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = 0.75 \text{ S}$$

By current division,

$$-\mathbf{I}_2 = \frac{4}{4+2}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1, \qquad \mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-\frac{2}{3}\mathbf{I}_1}{\frac{4}{3}\mathbf{I}_1} = -0.5 \text{ S}$$

To get $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, short-circuit the input port and connect a current source $\mathbf{I}_2$ to the output port as in Fig. 2(b). The 4-Ω resistor is short-circuited so that the 2-Ω and 8-Ω resistors are in parallel.

$$\mathbf{V}_2 = \mathbf{I}_2(8 \parallel 2) = \frac{8}{5}\mathbf{I}_2, \qquad \mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = \frac{5}{8} = 0.625 \text{ S}$$

By current division,

$$-\mathbf{I}_1 = \frac{8}{8+2}\mathbf{I}_2 = \frac{4}{5}\mathbf{I}_2, \qquad \mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\frac{4}{5}\mathbf{I}_2}{\frac{8}{5}\mathbf{I}_2} = -0.5 \text{ S}$$

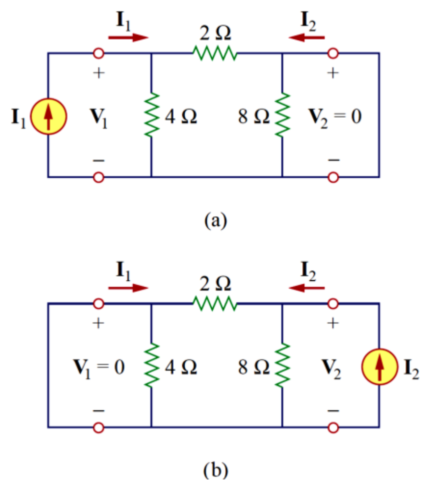


Fig.2
(a) finding y_{11} &y_{21}
(b) finding y_{12} and y_{22}

■ ,(**METHOD 2** Alternatively, comparing Fig. 1 with Fig. 3.2(a

$$\mathbf{y}_{12} = -\frac{1}{2}\,\mathrm{S} = \mathbf{y}_{21}$$

$$\mathbf{y}_{11} + \mathbf{y}_{12} = \frac{1}{4} \quad \Rightarrow \quad \mathbf{y}_{11} = \frac{1}{4} - \mathbf{y}_{12} = 0.75\ \mathrm{S}$$

$$\mathbf{y}_{22} + \mathbf{y}_{12} = \frac{1}{8} \quad \Rightarrow \quad \mathbf{y}_{22} = \frac{1}{8} - \mathbf{y}_{12} = 0.625\ \mathrm{S}$$

as obtained previously.

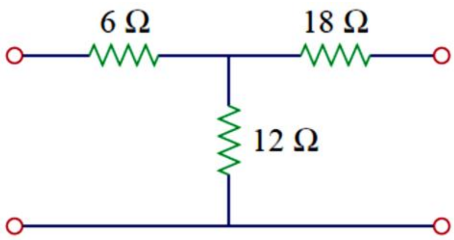
H.W.3: Obtain the *y* parameters for the *T* network shown in Fig.

Answer:

$\mathbf{y}_{11} = 75.77$ mS,

$\mathbf{y}_{12} = \mathbf{y}_{21} = -30.3$ mS,

$\mathbf{y}_{22} = 45.47$ mS.



Example 4: Determine the *y* parameters for the two-port shown

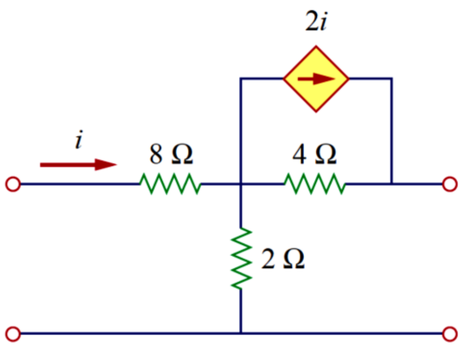
Solution: We follow the same procedure as in the previous example. To get $\mathbf{y}_{11}$-and $\mathbf{y}_{21}$, we use the circuit in Fig. 2 (a), in which port 2 is short circuited and a current source is applied to port 1. At node 1,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - 0}{4}$$

But $\mathbf{I}_1 = \dfrac{\mathbf{V}_1 - \mathbf{V}_o}{8}$; therefore,

$$0 = \frac{\mathbf{V}_1 - \mathbf{V}_o}{8} + \frac{3\mathbf{V}_o}{4}$$

$$0 = \mathbf{V}_1 - \mathbf{V}_o + 6\mathbf{V}_o \quad \Rightarrow \quad \mathbf{V}_1 = -5\mathbf{V}_o$$



. **Fig. 1**

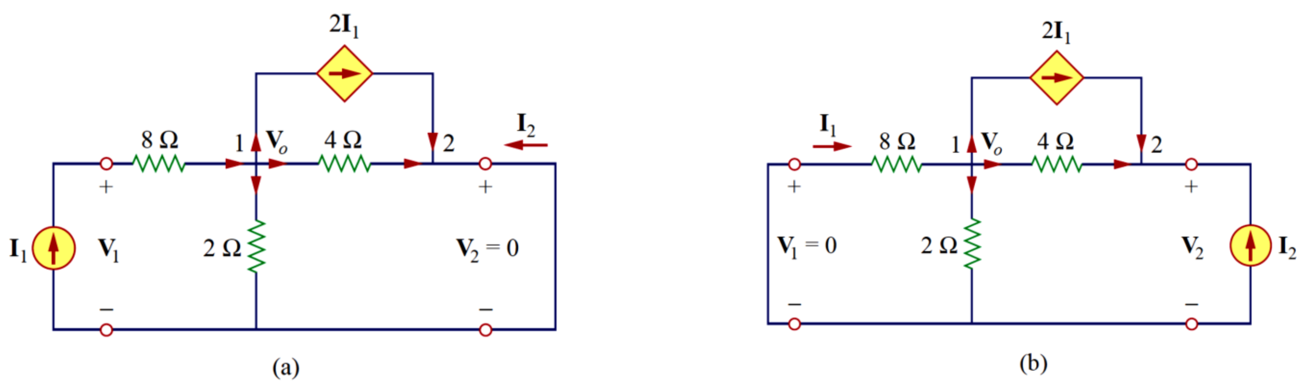


Fig.2 (a) finding y_{11} &y_{21} (b) finding y_{12} and y_{22}

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_o - 1.5\mathbf{V}_o = -1.25\mathbf{V}_o$$

Hence,

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{1.25\mathbf{V}_o}{-5\mathbf{V}_o} = -0.25 \text{ S}$$

Similarly, we get $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$,using Fig. 2(b). At node 1

$$\frac{0 - \mathbf{V}_o}{8} = 2\mathbf{I}_1 + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

But $\mathbf{I}_1 = \dfrac{0 - \mathbf{V}_o}{8}$; therefore,

$$0 = -\frac{\mathbf{V}_o}{8} + \frac{\mathbf{V}_o}{2} + \frac{\mathbf{V}_o - \mathbf{V}_2}{4}$$

or

$$0 = -\mathbf{V}_o + 4\mathbf{V}_o + 2\mathbf{V}_o - 2\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 2.5\mathbf{V}_o$$

Hence,

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{V}_o/8}{2.5\mathbf{V}_o} = -0.05 \text{ S}$$

At node 2,

$$\frac{\mathbf{V}_o - \mathbf{V}_2}{4} + 2\mathbf{I}_1 + \mathbf{I}_2 = 0$$

or

$$-\mathbf{I}_2 = 0.25\mathbf{V}_o - \frac{1}{4}(2.5\mathbf{V}_o) - \frac{2\mathbf{V}_o}{8} = -0.625\mathbf{V}_o$$

Thus,

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{0.625\mathbf{V}_o}{2.5\mathbf{V}_o} = 0.25 \text{ S}$$

Notice that $\mathbf{y}_{12} \neq \mathbf{y}_{21}$ in this case, since the network is not reciprocal.

H.W.4: Obtain the *y* parameters for the circuit in Fig.

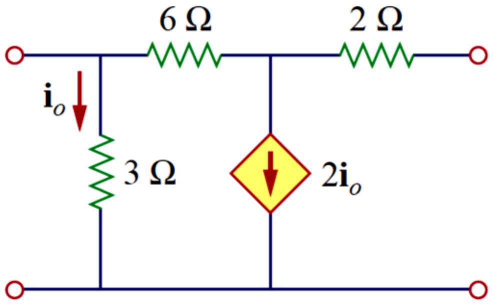
Answer:

$\mathbf{y}_{11} = 0.625$ S,

$\mathbf{y}_{12} = -0.125$ S,

$\mathbf{y}_{21} = 0.375$ S,

$\mathbf{y}_{22} = 0.125$ S.



3) Hybrid Parameters (h-parameters)

The ***z*** and ***y*** parameters of a two-port network do not always exist. So there is a need for developing another set of parameters. This third set of parameters is based on making V_1 and I_2 the dependent variables. Thus, we obtain

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \end{aligned} \qquad (4.1)$$

or in matrix form

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad (4.2)$$

The ***h*** terms are known as the ***hybrid parameters*** (or, ***h parameters***) because they are a hybrid combination of ratios. They are very useful for describing electronic devices such as transistors; it is much easier to measure experimentally the ***h*** parameters of such devices than to measure their ***z*** or ***y*** parameters.

In fact, we have seen that the ideal transformer in Fig. 2.5, described by Eq. (2.5), does not have ***z*** parameters. The ideal transformer can be described by the hybrid parameters, because Eq. (2.5) conforms with Eq. (4.1). The values of the parameters are determined as

$$\mathbf{h}_{11} = \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{12} = \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0}$$
$$\mathbf{h}_{21} = \left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_2=0}, \qquad \mathbf{h}_{22} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{I}_1=0} \qquad (4.3)$$

The parameters $\mathbf{h_{11}}$, $\mathbf{h_{12}}$, $\mathbf{h_{21}}$ and $\mathbf{h_{22}}$, represent an impedance, a voltage gain, a current gain, and an admittance, respectively. This is why they are called the hybrid parameters.

$$\begin{aligned} \mathbf{h}_{11} &= \text{Short-circuit input impedance} \\ \mathbf{h}_{12} &= \text{Open-circuit reverse voltage gain} \\ \mathbf{h}_{21} &= \text{Short-circuit forward current gain} \\ \mathbf{h}_{22} &= \text{Open-circuit output admittance} \end{aligned} \qquad (4.4)$$

The procedure for calculating the ***h*** parameters is similar to that used for the ***z*** or ***y*** parameters. We apply a voltage or current source to the appropriate port, short-circuit or open-circuit the other port, depending on the parameter of interest, and perform regular circuit analysis. For reciprocal networks, $\mathbf{h_{12}} = -\mathbf{h_{12}}$. This can be proved in the same way as we proved that $\mathbf{z_{12}} = \mathbf{z_{12}}$. Fig. 4.1 shows the hybrid model of a two-port network.

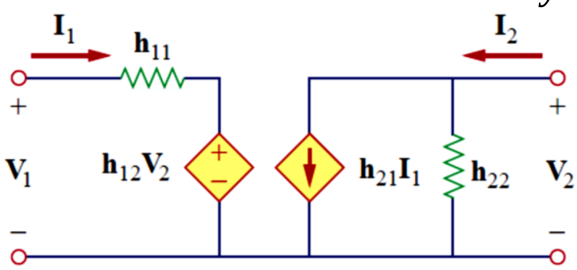


Fig. 4.1 The *h*-parameter equivalent network of a two-port network.

4) Inverse Hybrid Parameters (g-parameters)

A set of parameters closely related to the *h* parameters are the ***g parameters*** or ***inverse hybrid parameters.*** These are used to describe the terminal currents and voltages as

$$\boxed{\begin{aligned} \mathbf{I}_1 &= \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \end{aligned}} \qquad (5.1)$$

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_{11} & \mathbf{g}_{12} \\ \mathbf{g}_{21} & \mathbf{g}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{g}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} \qquad (5.2)$$

The values of the ***g*** parameters are determined as

$$\boxed{\begin{aligned} \mathbf{g}_{11} &= \frac{\mathbf{I}_1}{\mathbf{V}_1}\bigg|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2}\bigg|_{\mathbf{V}_1=0} \\ \mathbf{g}_{21} &= \frac{\mathbf{V}_2}{\mathbf{V}_1}\bigg|_{\mathbf{I}_2=0}, \qquad \mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}\bigg|_{\mathbf{V}_1=0} \end{aligned}} \qquad (5.3)$$

Thus, the inverse hybrid parameters are specifically called

$$\begin{aligned} \mathbf{g}_{11} &= \text{Open-circuit input admittance} \\ \mathbf{g}_{12} &= \text{Short-circuit reverse current gain} \\ \mathbf{g}_{21} &= \text{Open-circuit forward voltage gain} \\ \mathbf{g}_{22} &= \text{Short-circuit output impedance} \end{aligned} \qquad (5.4)$$

Fig. 5.1 shows the inverse hybrid model of a two-port network. ***The g parameters are frequently used to model field-effect transistors.***

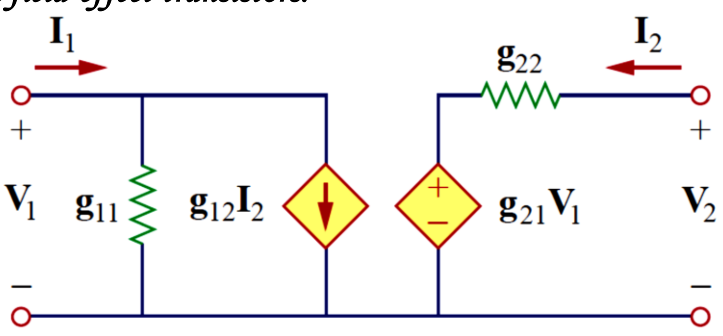


Fig. 5.1 The *g*-parameter model of a two-port network.

Example 5: Find the hybrid parameters for the two-port network of Fig. 1

Solution:

To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we short-circuit the output port and connect a current source $\mathbf{I}_1$ to the input port as shown in Fig.2(a).

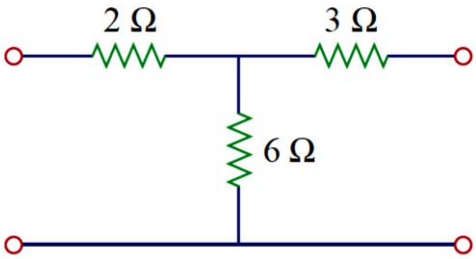


Fig.1

∴ From Fig. 2(a),

$$\mathbf{V}_1 = \mathbf{I}_1(2 + 3 \parallel 6) = 4\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = 4\ \Omega$$

Also, from Fig. 2(a) we obtain, by current division,

$$-\mathbf{I}_2 = \frac{6}{6 + 3}\mathbf{I}_1 = \frac{2}{3}\mathbf{I}_1$$

Hence,

$$\mathbf{h}_{21} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = -\frac{2}{3}$$

To obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, we open-circuit the input port and connect a voltage source $\mathbf{V}_2$ to the output port as in Fig.2(b). By voltage division,

$$\mathbf{V}_1 = \frac{6}{6 + 3}\mathbf{V}_2 = \frac{2}{3}\mathbf{V}_2$$

Hence,

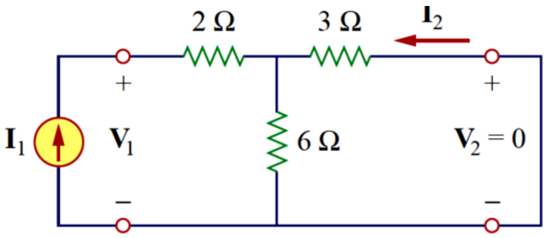
$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{2}{3}$$

Also,

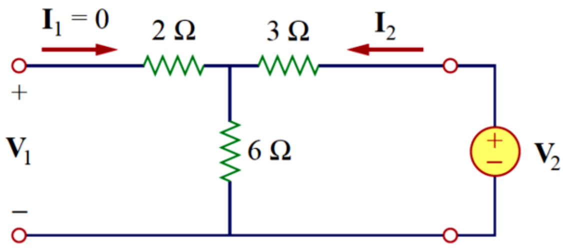
$$\mathbf{V}_2 = (3 + 6)\mathbf{I}_2 = 9\mathbf{I}_2$$

Thus,

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{9}\ \text{S}$$



(a)



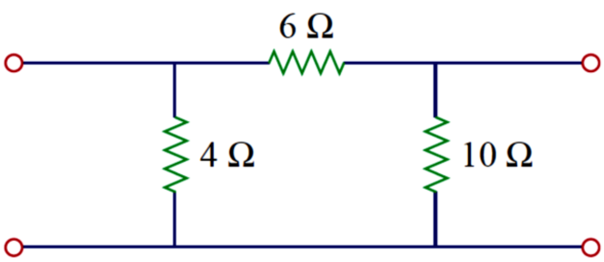
(b)

Fig. 2 (a) computing $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$,
(b) computing $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$.

H.W.5: Determine the *h* parameters for the circuit in Fig.

Answer:

$$\mathbf{h}_{11} = 2.4\ \Omega,$$
$$\mathbf{h}_{12} = 0.4,$$
$$\mathbf{h}_{21} = -0.4,$$
$$\mathbf{h}_{22} = 200\ \text{mS}.$$



Example 6: Determine the Thevenin equivalent at the output port of the circuit inFig.

Solution:

To find Z_{Th} and V_{Th}, we apply the normal procedure, keeping in mind the formulas relating the input and output ports of the ***h*** model. To obtain Z_{Th} , remove the 60-V voltage source at the input port and apply a 1-V voltage source at the output port, as shown in Fig. 2(a). From Eq. (4.1),

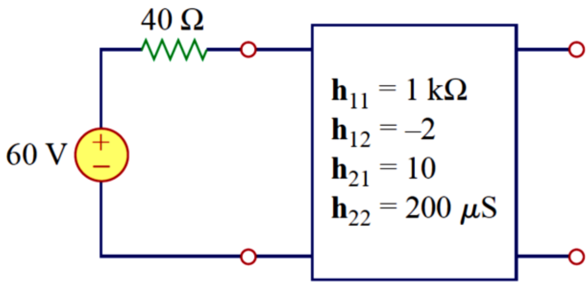


Fig. 1

$$V_1 = h_{11}I_1 + h_{12}V_2 \qquad (1)$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \qquad (2)$$

But $V_2 = 1$, and $V_1 = -40I_1$Substituting these into Eqs. (1) and . we get ,(2)

$$-40I_1 = h_{11}I_1 + h_{12} \quad \Rightarrow \quad I_1 = -\frac{h_{12}}{40 + h_{11}} \qquad (3)$$

$$I_2 = h_{21}I_1 + h_{22} \qquad (4)$$

Substituting Eq. (3) into Eq. (4)gives

$$I_2 = h_{22} - \frac{h_{21}h_{12}}{h_{11} + 40} = \frac{h_{11}h_{22} - h_{21}h_{12} + h_{22}40}{h_{11} + 40}$$

Therefore,

$$Z_{Th} = \frac{V_2}{I_2} = \frac{1}{I_2} = \frac{h_{11} + 40}{h_{11}h_{22} - h_{21}h_{12} + h_{22}40}$$

Substituting the values of the *h* parameters,

$$Z_{Th} = \frac{1000 + 40}{10^3 \times 200 \times 10^{-6} + 20 + 40 \times 200 \times 10^{-6}}$$

$$= \frac{1040}{20.21} = 51.46\ \Omega$$

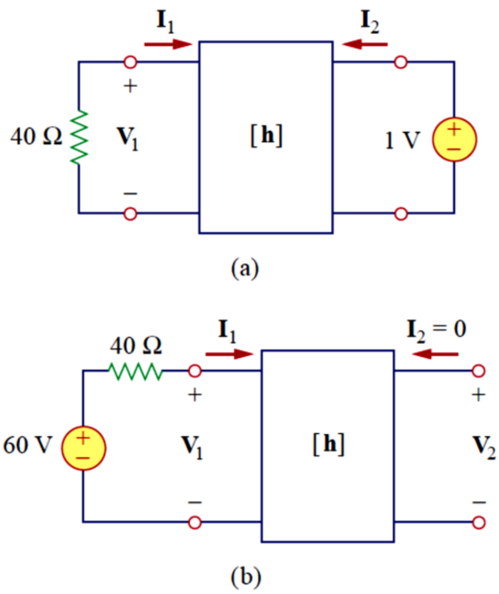


Fig. 2 (a) finding Z_{Th}, (b) finding V_{Th},

To get V_{Th} , we find the open-circuit voltage V_2 in Fig. 2(b). At the input port,

$$-60 + 40I_1 + V_1 = 0 \quad \Rightarrow \quad V_1 = 60 - 40I_1 \qquad (5)$$

At the output,

$\mathbf{I}_2 = 0$ **(6)**

Substituting Eqs. (5) and (6) into Eqs. (1) and (2), we obtain

$$60 - 40\mathbf{I}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

or

$$60 = (\mathbf{h}_{11} + 40)\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \qquad \textbf{(7)}$$

and

$$0 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{I}_1 = -\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}}\mathbf{V}_2 \qquad \textbf{(8)}$$

Now substituting Eq. (8) into Eq. (7) gives

$$60 = \left[-(\mathbf{h}_{11} + 40)\frac{\mathbf{h}_{22}}{\mathbf{h}_{21}} + \mathbf{h}_{12}\right]\mathbf{V}_2$$

or

$$\mathbf{V}_{\text{Th}} = \mathbf{V}_2 = \frac{60}{-(\mathbf{h}_{11} + 40)\mathbf{h}_{22}/\mathbf{h}_{21} + \mathbf{h}_{12}} = \frac{60\mathbf{h}_{21}}{\mathbf{h}_{12}\mathbf{h}_{21} - \mathbf{h}_{11}\mathbf{h}_{22} - 40\mathbf{h}_{22}}$$

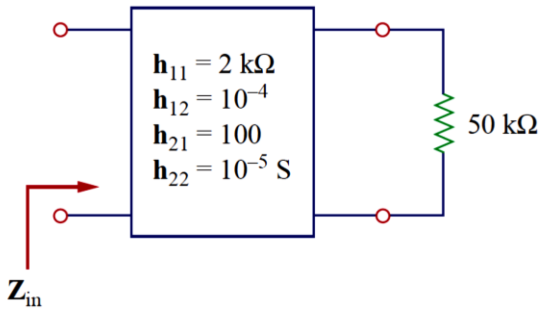
Substituting the values of the h parameters,

$$\mathbf{V}_{\text{Th}} = \frac{60 \times 10}{-20.21} = -29.69 \text{ V}$$

H.W.6: Find the impedance at the input port of the circuit in Fig.

Answer:

1.6667 kΩ.



Example 7: Find the ***g*** parameters as functions of ***s*** for the circuit in Fig.

Solution:

In the s domain,

$$1\text{ H} \quad \Rightarrow \quad sL = s, \qquad 1\text{ F} \quad \Rightarrow \quad \frac{1}{sC} = \frac{1}{s}$$

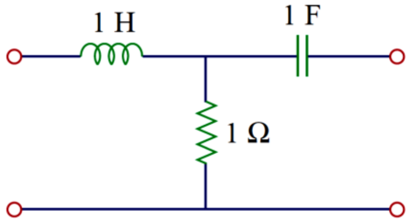


Fig. 1

To get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$, we open-circuit the output port and connect a ,voltage source $\mathbf{V}_1$ to the input port as in Fig. 2(a). From the figure

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{s + 1}$$

or

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{s+1}$$

By voltage division,

$$\mathbf{V}_2 = \frac{1}{s+1}\mathbf{V}_1$$

or

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s+1}$$

To obtain $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$, we short-circuit the input port and connect a curren ,source $\mathbf{I}_2$ to the output port as in Fig.2(b). By current division

$$\mathbf{I}_1 = -\frac{1}{s+1}\mathbf{I}_2$$

or

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = -\frac{1}{s+1}$$

Also,

$$\mathbf{V}_2 = \mathbf{I}_2\left(\frac{1}{s} + s \parallel 1\right)$$

or

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{1}{s} + \frac{s}{s+1} = \frac{s^2+s+1}{s(s+1)}$$

Thus,

$$[\mathbf{g}] = \begin{bmatrix} \frac{1}{s+1} & -\frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$$

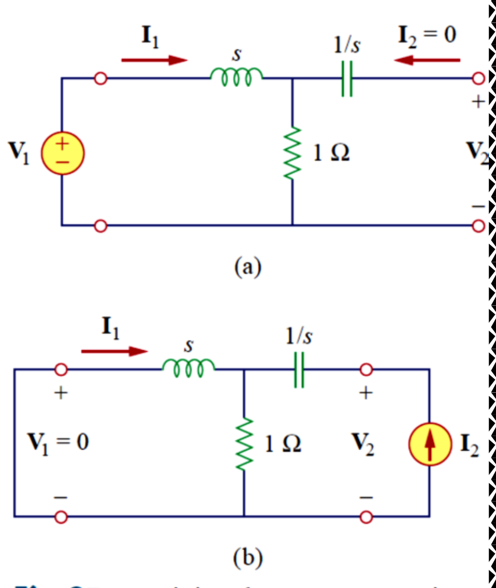
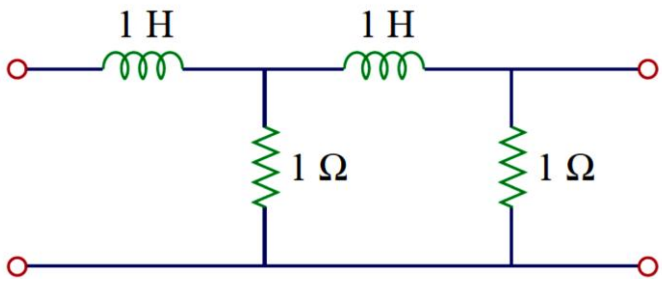


Fig.2 Determining the *g* parameters in the *s* domain

H.W.7: For the ladder network in Fig., determine the ***g*** parameters in the ***s*** domain.

Answer:

$$[\mathbf{g}] = \begin{bmatrix} \frac{s+2}{s^2+3s+1} & -\frac{1}{s^2+3s+1} \\ \frac{1}{s^2+3s+1} & \frac{s(s+2)}{s^2+3s+1} \end{bmatrix}$$



5) *Transmission Parameters (ABCD parameters)*

Since there are no restrictions on which terminal voltages and currents should be considered independent and which should be dependent variables, we expect to be able to generate many sets of parameters. Another set of parameters relates the variables at the input port to those at the output port. Thus,

$$\boxed{\begin{aligned} \mathbf{V_1} &= \mathbf{AV_2} - \mathbf{BI_2} \\ \mathbf{I_1} &= \mathbf{CV_2} - \mathbf{DI_2} \end{aligned}} \qquad \textbf{(6.1)}$$

or

$$\begin{bmatrix} \mathbf{V_1} \\ \mathbf{I_1} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V_2} \\ -\mathbf{I_2} \end{bmatrix} = [\mathbf{T}] \begin{bmatrix} \mathbf{V_2} \\ -\mathbf{I_2} \end{bmatrix} \qquad \textbf{(6.2)}$$

Equations (6.1) and (6.2) relate the input variables ($\mathbf{V_1}$ and $\mathbf{I_1}$) to the output variables ($\mathbf{V_2}$ and $-\mathbf{I_2}$). Notice that in computing the transmission parameters $-\mathbf{I_2}$, is used rather than $\mathbf{I_2}$, because the current is considered to be leaving the network, as shown in Fig. 6.1, as opposed to entering the network as in Fig. 1.1(b). This is done merely for conventional reasons; when you cascade two-ports (output to input), it is most logical to think of $\mathbf{I_2}$ as leaving the two-port. It is also customary in the power industry to consider $\mathbf{I_2}$ as leaving the two-port.

The transmission parameters are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables ($\mathbf{V_1}$ and $\mathbf{I_1}$) in terms of the receiving-end variables ($\mathbf{V_2}$ and $-\mathbf{I_2}$). For this reason, they are called ***transmission parameters***. They are also known as **ABCD** parameters. They are used in the design of telephone systems, microwave networks, and radars.

The transmission parameters are determined as

$$\boxed{\begin{aligned} \mathbf{A} &= \left.\frac{\mathbf{V_1}}{\mathbf{V_2}}\right|_{\mathbf{I_2}=0}, \qquad \mathbf{B} = -\left.\frac{\mathbf{V_1}}{\mathbf{I_2}}\right|_{\mathbf{V_2}=0} \\ \mathbf{C} &= \left.\frac{\mathbf{I_1}}{\mathbf{V_2}}\right|_{\mathbf{I_2}=0}, \qquad \mathbf{D} = -\left.\frac{\mathbf{I_1}}{\mathbf{I_2}}\right|_{\mathbf{V_2}=0} \end{aligned}} \qquad \textbf{(6.3)}$$

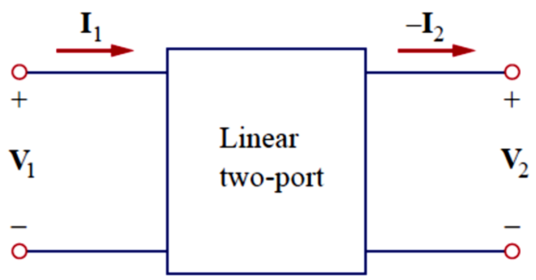


Fig. 6.1

Thus, the transmission parameters are called, specifically,

$$\begin{aligned} \mathbf{A} &= \text{Open-circuit voltage ratio} \\ \mathbf{B} &= \text{Negative short-circuit transfer impedance} \\ \mathbf{C} &= \text{Open-circuit transfer admittance} \\ \mathbf{D} &= \text{Negative short-circuit current ratio} \end{aligned} \qquad \textbf{(6.4)}$$

A and **D** are dimensionless, **B** is in ohms, and **C** is in siemens. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

6) *Inverse Transmission Parameters (t parameters)*

Our last set of parameters may be defined by expressing the variables at the output port in terms of the variables at the input port. We obtain

$$\begin{aligned} \mathbf{V}_2 &= \mathbf{a}\mathbf{V}_1 - \mathbf{b}\mathbf{I}_1 \\ \mathbf{I}_2 &= \mathbf{c}\mathbf{V}_1 - \mathbf{d}\mathbf{I}_1 \end{aligned} \quad (7.1)$$

or

$$\begin{bmatrix} \mathbf{V}_2 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} = [\mathbf{t}] \begin{bmatrix} \mathbf{V}_1 \\ -\mathbf{I}_1 \end{bmatrix} \quad (7.2)$$

The parameters **a**, **b**, **c**, and **d** are called the ***inverse transmission***, or ***t, parameters***. They are determined as follows:

$$\mathbf{a} = \left.\frac{\mathbf{V}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, \qquad \mathbf{b} = -\left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0}$$

$$\mathbf{c} = \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{I}_1=0}, \qquad \mathbf{d} = -\left.\frac{\mathbf{I}_2}{\mathbf{I}_1}\right|_{\mathbf{V}_1=0} \quad (7.3)$$

The inverse transmission parameters are known individually as

$\mathbf{a}$ = Open-circuit voltage gain
$\mathbf{b}$ = Negative short-circuit transfer impedance
$\mathbf{c}$ = Open-circuit transfer admittance
$\mathbf{d}$ = Negative short-circuit current gain
(7.4)

While **a** and **d** are dimensionless, **b** and **c** are in ohms and siemens, respectively.
In terms of the transmission or inverse transmission parameters, a network is reciprocal if

$$\mathbf{AD} - \mathbf{BC} = 1, \qquad \mathbf{ad} - \mathbf{bc} = 1 \quad (7.5)$$

These relations can be proved in the same way as the transfer impedance relations for the *z* parameters.

Example 8: Find the transmission parameters for the two-port network in Fig.

Solution:

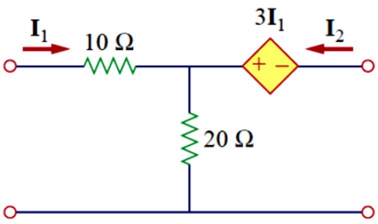


Fig. 1

:

To determine **A** and **C**, we leave the output port open as in Fig. 2(a),so that $\mathbf{I}_2 = 0$ and place a voltage source $\mathbf{V}_1$ at the input port. We have

$$\mathbf{V}_1 = (10 + 20)\mathbf{I}_1 = 30\mathbf{I}_1 \quad \text{and} \quad \mathbf{V}_2 = 20\mathbf{I}_1 - 3\mathbf{I}_1 = 17\mathbf{I}_1$$

Thus,

$$\mathbf{A} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{30\mathbf{I}_1}{17\mathbf{I}_1} = 1.765, \qquad \mathbf{C} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_1}{17\mathbf{I}_1} = 0.0588 \text{ S}$$

To obtain **B** and **D**, we short-circuit the output port so that $\mathbf{V}_2 = 0$ as shown in Fig. 2 (b) and place a voltage source $\mathbf{V}_1$ at the input port. At node *a* in the circuit of Fig. 2(b), KCL gives

$$\frac{\mathbf{V}_1 - \mathbf{V}_a}{10} - \frac{\mathbf{V}_a}{20} + \mathbf{I}_2 = 0 \qquad \textbf{(1)}$$

But $\mathbf{V}_a = 3\mathbf{I}_1$ and $\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_a)/10$. Combining these gives

$$\mathbf{V}_a = 3\mathbf{I}_1 \qquad \mathbf{V}_1 = 13\mathbf{I}_1 \qquad \textbf{(2),}$$

Substituting $\mathbf{V}_a = 3\mathbf{I}_1$ into Eq. (1) and replacing the first term with $\mathbf{I}_1$,

$$\mathbf{I}_1 - \frac{3\mathbf{I}_1}{20} + \mathbf{I}_2 = 0 \quad \Rightarrow \quad \frac{17}{20}\mathbf{I}_1 = -\mathbf{I}_2$$

Therefore,

$$\mathbf{D} = -\frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{20}{17} = 1.176, \qquad \mathbf{B} = -\frac{\mathbf{V}_1}{\mathbf{I}_2} = \frac{-13\mathbf{I}_1}{(-17/20)\mathbf{I}_1} = 15.29\ \Omega$$

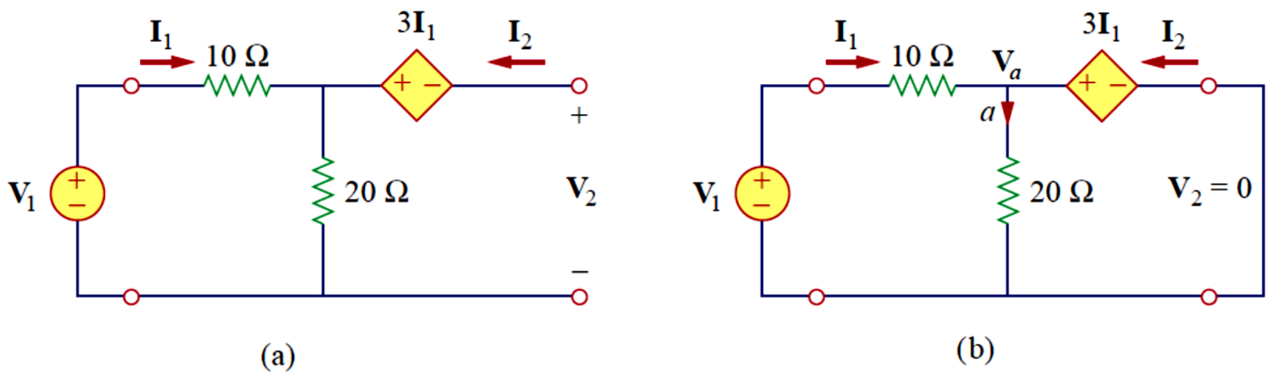


Fig. 2 (a) finding **A** and **C**, (b) finding **B** and **D**

H.W.8: Find the transmission parameters for the circuit in Fig. shown in **H.W.3**

Answer:

$\mathbf{A} = 1.5$, $\mathbf{B} = 22\ \Omega$, $\mathbf{C} = 125$ mS, $\mathbf{D} = 2.5$.

Example 9: The **ABCD** parameters of the two-port network in Fig. 1 are

$$\begin{bmatrix} 4 & 20\Omega \\ 0.1S & 2 \end{bmatrix}$$

The output port is connected to a variable load for maximum power transfer. Find R_L and the maximum power transferred.

Solution:

What we need is to find the Thevenin equivalent ($\mathbf{Z}_{Th}$ and $\mathbf{V}_{Th}$) at the load or output port. We find $\mathbf{Z}_{Th}$ using the circuit in Fig. 2(a). Our goal is to get $\mathbf{Z}_{Th} = \mathbf{V}_2/\mathbf{I}_2$. Substituting the given **ABCD** parameters into Eq. (2), we obtain

$$\mathbf{V}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2 \qquad \textbf{(1)}$$

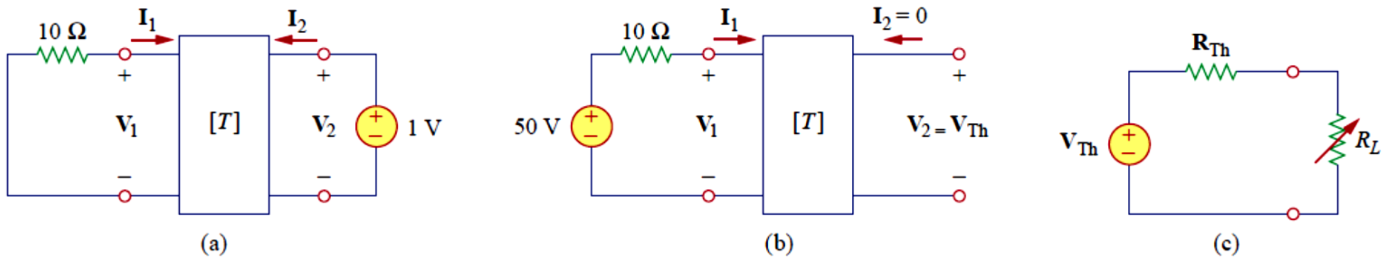
$$\mathbf{I}_1 = 0.1\mathbf{V}_2 - 2\mathbf{I}_2 \qquad \textbf{(2)}$$

At the input port, $\mathbf{V}_1 = -10\mathbf{I}_1$Substituting this into Eq. (1) gives .

$$-10\mathbf{I}_1 = 4\mathbf{V}_2 - 20\mathbf{I}_2$$

or

$$\mathbf{I}_1 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2 \qquad \textbf{(3)}$$



.**Fig.2** (a) finding $\mathbf{Z}_{Th}$. (b) finding $\mathbf{V}_{Th}$. (c) finding R_L for maximum power transfer

,Setting the right-hand sides of Eqs. (2) and (3) equal

$$0.1\mathbf{V}_2 - 2\mathbf{I}_2 = -0.4\mathbf{V}_2 + 2\mathbf{I}_2 \quad \Rightarrow \quad 0.5\mathbf{V}_2 = 4\mathbf{I}_2$$

Hence,

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{4}{0.5} = 8\ \Omega$$

To find $\mathbf{V}_{Th}$, we use the circuit in Fig. 2 (b). At the output port $\mathbf{I}_2 = 0$ and at the input port $\mathbf{V}_1 = 50 - 10\mathbf{I}_1$. (Substituting these into Eqs. (1) ,and (2)

$$50 - 10\mathbf{I}_1 = 4\mathbf{V}_2 \qquad \textbf{(4)}$$

$$\mathbf{I}_1 = 0.1\mathbf{V}_2 \qquad \textbf{(5)}$$

,Substituting Eq. (5) into Eq. (4)

$$50 - \mathbf{V}_2 = 4\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_2 = 10$$

Thus,

$\mathbf{V}_{Th} = \mathbf{V}_2 = 10$ V

,The equivalent circuit is shown in Fig. 2(c). For maximum power transfer

$$R_L = \mathbf{Z}_{\text{Th}} = 8\ \Omega$$

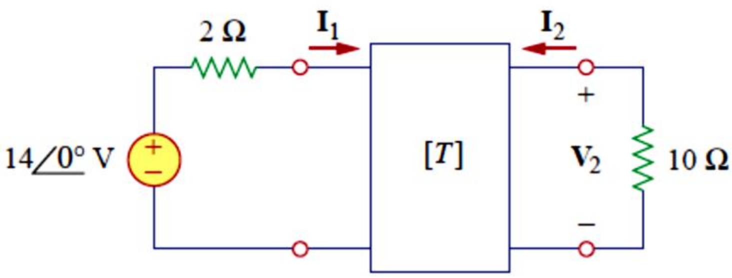
The maximum power is

$$P = I^2 R_L = \left(\frac{\mathbf{V}_{\text{Th}}}{2R_L}\right)^2 R_L = \frac{\mathbf{V}_{\text{Th}}^2}{4R_L} = \frac{100}{4 \times 8} = 3.125\ \text{W}$$

H.W.9: Find $\mathbf{I_1}$ and $\mathbf{I_2}$ if the transmission parameters for the two-port in Fig.

$$\begin{bmatrix} 5 & 10\Omega \\ 0.4S & 1 \end{bmatrix}$$

Answer:
1 A, - 0.2 A.



7) Relationships Between Parameters

Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated.

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} \qquad \textbf{(8.1)}$$

or

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}]^{-1} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad \textbf{(8.2)}$$

,Also, from Eq. (5.2)

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{y}] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad \textbf{(8.3)}$$

Comparing Eqs. (8.2) and (8.3), we see that

$$[\mathbf{y}] = [\mathbf{z}]^{-1} \qquad \textbf{(8.4)}$$

The adjoint of the [z] matrix is

$$\begin{bmatrix} \mathbf{z}_{22} & -\mathbf{z}_{12} \\ -\mathbf{z}_{21} & \mathbf{z}_{11} \end{bmatrix}$$

and its determinant is

$$_z = \mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}\Delta$$

Substituting these intoEq. (8.4), we get

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \frac{\begin{bmatrix} \mathbf{z}_{22} & -\mathbf{z}_{12} \\ -\mathbf{z}_{21} & \mathbf{z}_{11} \end{bmatrix}}{\Delta_z} \qquad \textbf{(8.5)}$$

Equating terms yields

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z}, \qquad \mathbf{y}_{12} = -\frac{\mathbf{z}_{12}}{\Delta_z}, \qquad \mathbf{y}_{21} = -\frac{\mathbf{z}_{21}}{\Delta_z}, \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} \qquad \textbf{(8.6)}$$

As a second example, let us determine the h parameters from the z ,parameters. From Eq. (2.1)

$$\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \qquad \textbf{(8.7a)}$$

$$\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \qquad \textbf{(8.7b)}$$

,Making $\mathbf{I}_2$ the subject of Eq. (8.7b)

$$\mathbf{I}_2 = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{z}_{22}}\mathbf{V}_2 \qquad \textbf{(8.8)}$$

,Substituting this into Eq. (8.7a)

$$\mathbf{V}_1 = \frac{\mathbf{z}_{11}\mathbf{z}_{22} - \mathbf{z}_{12}\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}\mathbf{V}_2 \qquad \textbf{(8.9)}$$

,Putting Eqs. (8.8) and (8.9) in matrix form

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\Delta_z}{\mathbf{z}_{22}} & \dfrac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ -\dfrac{\mathbf{z}_{21}}{\mathbf{z}_{22}} & \dfrac{1}{\mathbf{z}_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad \textbf{(8.10)}$$

,From Eq. (4.2)

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Comparing this with Eq.(8.10), we obtain

$$\mathbf{h}_{11} = \frac{\Delta_z}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{12} = \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{21} = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}, \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{z}_{22}} \qquad \textbf{(8.11)}$$

Table 8.1 provides the conversion formulas for the six sets of two-port parameters. Given one set of parameters, Table 8.1 can be used to find other parameters. For example, given the T parameters, we find the . corresponding h parameters in the fifth column of the third row

TABLE 8.1

Conversion of two-port parameters.

	z		y		h		g		T		t	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$

$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$

Also, given that $\mathbf{z}_{21} = \mathbf{z}_{12}$ for a reciprocal network, we can use the table to express this condition in terms of other parameters. It can also be shown that

$$[\mathbf{g}] = [\mathbf{h}]^{-1} \tag{8.12}$$

but

$$[\mathbf{t}] \neq [\mathbf{T}]^{-1} \tag{8.13}$$

Example 10: Find $[z]$ and $[g]$ of a two-port network if

$$[T] = \begin{bmatrix} 10 & 1.5\Omega \\ 2\,S & 4 \end{bmatrix}$$

Solution:

If $\mathbf{A} = 10$, $\mathbf{B} = 1.5$, $\mathbf{C} = 2$, $\mathbf{D} = 4$, the determinant of the matrix is

$$\Delta_T = \mathbf{AD} - \mathbf{BC} = 40 - 3 = 37$$

,From Table 8.1

$$\mathbf{z}_{11} = \frac{\mathbf{A}}{\mathbf{C}} = \frac{10}{2} = 5, \qquad \mathbf{z}_{12} = \frac{\Delta_T}{\mathbf{C}} = \frac{37}{2} = 18.5$$

$$\mathbf{z}_{21} = \frac{1}{\mathbf{C}} = \frac{1}{2} = 0.5, \qquad \mathbf{z}_{22} = \frac{\mathbf{D}}{\mathbf{C}} = \frac{4}{2} = 2$$

$$\mathbf{g}_{11} = \frac{\mathbf{C}}{\mathbf{A}} = \frac{2}{10} = 0.2, \qquad \mathbf{g}_{12} = -\frac{\Delta_T}{\mathbf{A}} = -\frac{37}{10} = -3.7$$

$$\mathbf{g}_{21} = \frac{1}{\mathbf{A}} = \frac{1}{10} = 0.1, \qquad \mathbf{g}_{22} = \frac{\mathbf{B}}{\mathbf{A}} = \frac{1.5}{10} = 0.15$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix}\Omega, \qquad [\mathbf{g}] = \begin{bmatrix} 0.2\text{ S} & -3.7 \\ 0.1 & 0.15\ \Omega \end{bmatrix}$$

H.W.10: Determine $[y]$ and $[T]$ of a two-port network whose z parameters are

$$[z] = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix}\Omega$$

Answer:

$$[\mathbf{y}] = \begin{bmatrix} 0.3 & -0.2 \\ -0.2 & 0.3 \end{bmatrix}\text{S}, \quad [\mathbf{T}] = \begin{bmatrix} 1.5 & 5\ \Omega \\ 0.25\text{ S} & 1.5 \end{bmatrix}.$$

Example 11: Obtain the *y* parameters of the op amp circuit in Fig. 1. Show that the circuit has no *z* parameters

Solution:

Since no current can enter the input terminals of the op amp, $\mathbf{I}_1 = 0$, which can be expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$ as

$$\mathbf{I}_1 = 0\mathbf{V}_1 + 0\mathbf{V}_2 \qquad \textbf{(1)}$$

Comparing this with Eq.(3.1) gives

$$\mathbf{y}_{11} = 0 = \mathbf{y}_{12}$$

Also,

$$\mathbf{V}_2 = R_3\mathbf{I}_2 + \mathbf{I}_o(R_1 + R_2)$$

where $\mathbf{I}_o$ is the current through R_1 and R_2. But $\mathbf{I}_o = \mathbf{V}_1/R_1$. Hence,

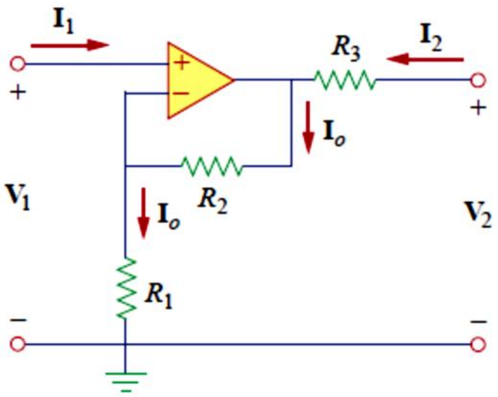
$$\mathbf{V}_2 = R_3\mathbf{I}_2 + \frac{\mathbf{V}_1(R_1 + R_2)}{R_1}$$

which can be written as

$$\mathbf{I}_2 = -\frac{(R_1 + R_2)}{R_1R_3}\mathbf{V}_1 + \frac{\mathbf{V}_2}{R_3}$$

Comparing this with Eq.(3.1) shows that

$$\mathbf{y}_{21} = -\frac{(R_1 + R_2)}{R_1R_3}, \qquad \mathbf{y}_{22} = \frac{1}{R_3}$$



. **Fig. 1**

The determinant of the [y] matrix is

$$\Delta_y = \mathbf{y}_{11}\mathbf{y}_{22} - \mathbf{y}_{12}\mathbf{y}_{21} = 0$$

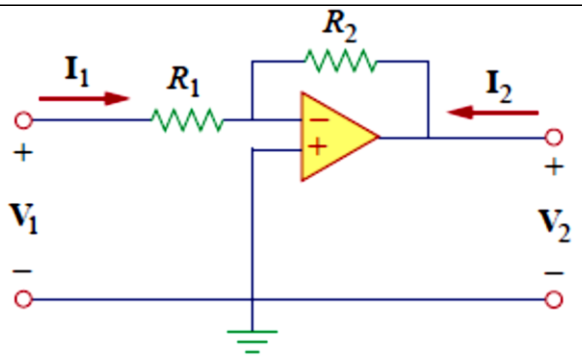
Since $\Delta_y = 0$, the [**y**] matrix has no inverse; therefore, the [**z**] matrix does not exist according to Eq. (8.4). Note that the circuit is not reciprocal because of the active element

H.W.11: Find the *z* parameters of the op amp circuit in Fig. Show that the circuit has no *y* parameters.

Answer:

$$[\mathbf{z}] = \begin{bmatrix} R_1 & 0 \\ -R_2 & 0 \end{bmatrix}$$

Since $[\mathbf{z}]^{-1}$ does not exist, [**y**] does not exist.



8) Interconnection of Networks

A large, complex network may be divided into subnetworks for the purposes of analysis and design. The subnetworks are modeled as twoport networks, interconnected to form the original network.

The interconnection can be in series, in parallel, or in cascade. Although the interconnected network can be described by any of the six parameter sets, a certain set of parameters may have a definite advantage. For example, when the networks are in series, their individual ***z*** parameters add up to give the ***z parameters*** of the larger network. When they are in parallel, their individual ***y parameters*** add up to give the ***y parameters*** of the larger network. When they are cascaded, their individual ***transmission parameters*** can be multiplied together to get the ***transmission parameters*** of the larger network.

8.1) series connection

Consider the two two-port networks (in Fig. 9.1.) The networks are in series because their input currents are the same and their voltages add. In addition, each network has a common reference, and when the circuits are placed in series, the common reference points of each circuit are connected together. For network N_a ,

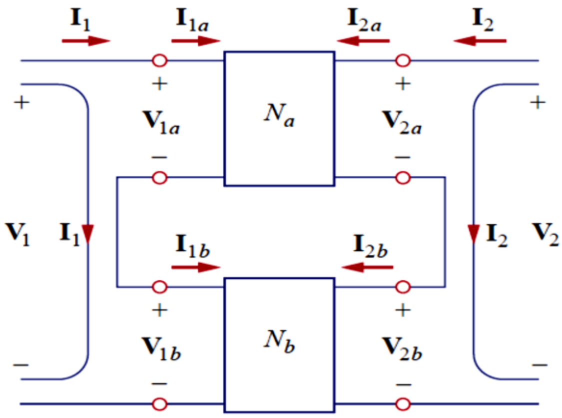


Fig. 9.1 Series connection of .two two-port networks

$$\mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{12a}\mathbf{I}_{2a}$$
$$\mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{22a}\mathbf{I}_{2a}$$

and for network N_b,

$$\mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12b}\mathbf{I}_{2b}$$
$$\mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22b}\mathbf{I}_{2b}$$

We notice from Fig.9.1 that

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}, \qquad \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}$$

and that

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a}$$
$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a}$$

Thus, the z parameters for the overall network are

$$\begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11a} + \mathbf{z}_{11b} & \mathbf{z}_{12a} + \mathbf{z}_{12b} \\ \mathbf{z}_{21a} + \mathbf{z}_{21b} & \mathbf{z}_{22a} + \mathbf{z}_{22b} \end{bmatrix} \qquad \textbf{(9.5)}$$

or

$$\boxed{[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]} \qquad \textbf{(9.6)}$$

z parameters for the overall network are the sum of the ***z parameters*** for the individual networks. This can be extended to ***n*** networks in series. If two two-port networks in the [**h**] model, for example, are connected in series, we use Table 8.1 to convert the **h** to **z** and then apply Eq. (9.6). We finally convert the result back to **h** using Table 8.1

8.2) parallel connection

Two two-port networks are in ***parallel*** when their port voltages are equal and the port currents of the larger network are the sums of the individual port currents. In addition, each circuit must have a common reference and when the networks are connected together, they must all have their common references tied together. The parallel connection of two two-port networks is shown in Fig. 9.2. For the two networks,

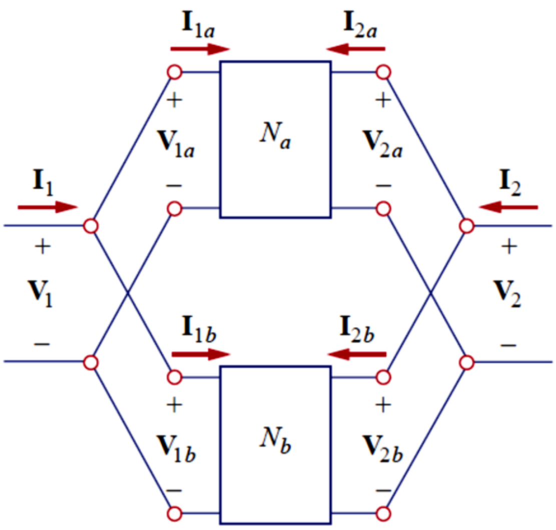


Fig. 9.2 Parallel connection of .two two-port networks

$$\begin{aligned} \mathbf{I}_{1a} &= \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{12a}\mathbf{V}_{2a} \\ \mathbf{I}_{2a} &= \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{22a}\mathbf{V}_{2a} \end{aligned} \qquad \textbf{(9.7)}$$

$$\begin{aligned} \mathbf{I}_{1b} &= \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12b}\mathbf{V}_{2b} \\ \mathbf{I}_{2a} &= \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22b}\mathbf{V}_{2b} \end{aligned} \qquad \textbf{(9.8)}$$

.2

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}, \qquad \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b} \qquad \textbf{(9.9a)}$$

$$\mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}, \qquad \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b} \qquad \textbf{(9.9b)}$$

Substituting Eqs. (9.7) and (9.8) into Eq. (9.9b) yields

$$\begin{aligned} \mathbf{I}_1 &= (\mathbf{y}_{11a} + \mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a} + \mathbf{y}_{12b})\mathbf{V}_2 \\ \mathbf{I}_2 &= (\mathbf{y}_{21a} + \mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a} + \mathbf{y}_{22b})\mathbf{V}_2 \end{aligned} \tag{9.10}$$

Thus, the y parameters for the overall network are

$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix} \tag{9.11}$$

or

$$\boxed{[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]} \tag{9.12}$$

showing that the *y parameters* of the overall network are the sum of the *y parameters* of the individual networks. The result can be extended to n two-port networks in parallel.

8.3) cascaded connection

Two networks are said to be ***cascaded*** when the output of one is the input of the other. The connection of two two-port networks in cascade is shown in Fig. 9.3. For the two networks,

$$\begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{I}_{1a} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_a & \mathbf{B}_a \\ \mathbf{C}_a & \mathbf{D}_a \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2a} \\ -\mathbf{I}_{2a} \end{bmatrix} \tag{9.13}$$

$$\begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{I}_{1b} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_b & \mathbf{B}_b \\ \mathbf{C}_b & \mathbf{D}_b \end{bmatrix} \begin{bmatrix} \mathbf{V}_{2b} \\ -\mathbf{I}_{2b} \end{bmatrix} \tag{9.14}$$

From Fig. 9.3

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1a} \\ \mathbf{I}_{1a} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{V}_{2a} \\ -\mathbf{I}_{2a} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{1b} \\ \mathbf{I}_{1b} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{V}_{2b} \\ -\mathbf{I}_{2b} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{bmatrix} \tag{9.15}$$

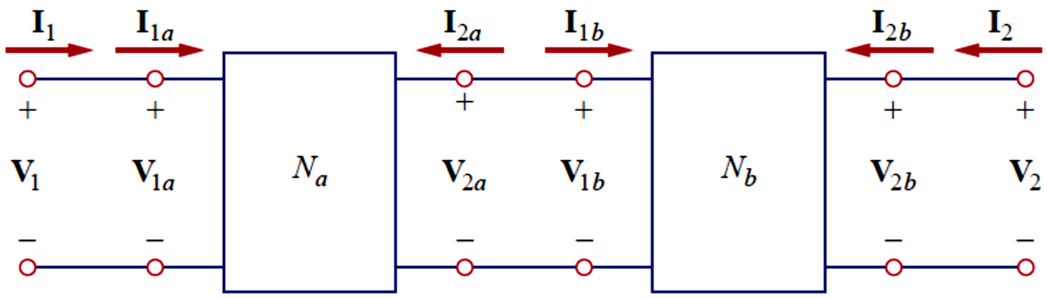


Figure 9.3 Cascade connection of two two-port networks

Substituting these into Eqs.(9.13) and (9.14),

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad (9.16)$$

Thus, the transmission parameters for the overall network are the product of the transmission parameters for the individual transmission parameters:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \qquad (9.17)$$

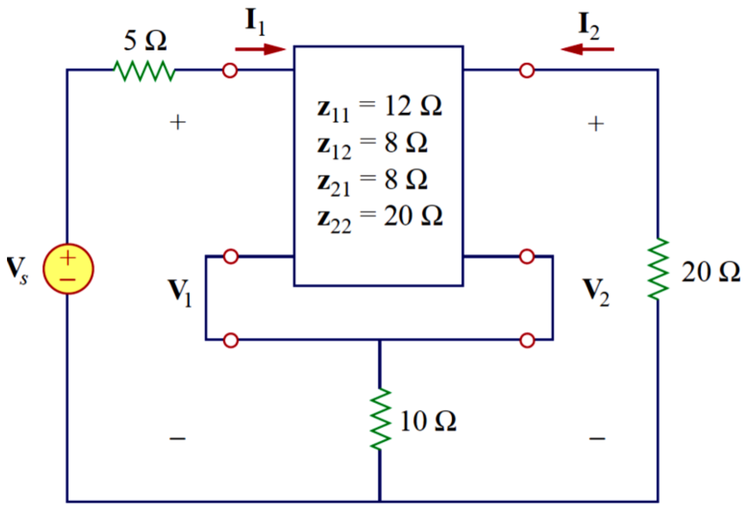
or

$$\boxed{[T] = [T_a][T_b]} \qquad (9.18)$$

It is this property that makes the transmission parameters so useful. Keep in mind that the multiplication of the matrices must be in the order in which the networks N_a and N_b are cascaded.

Example 12: Evaluate V_2 / V_s in the circuit in Fig.

Solution:



This may be regarded as two two-ports in series. For N_b,

$$z_{12b} = z_{21b} = 10 = z_{11b} = z_{22b}$$

Thus,

$$[z] = [z_a] + [z_b]$$

$$= \begin{bmatrix} 12 & 8 \\ 8 & 20 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 10 \end{bmatrix} = \begin{bmatrix} 22 & 18 \\ 18 & 30 \end{bmatrix}$$

But

$$V_1 = z_{11}I_1 + z_{12}I_2 = 22I_1 + 18I_2 \qquad (1)$$

$$V_2 = z_{21}I_1 + z_{22}I_2 = 18I_1 + 30I_2 \qquad (2)$$

Also, at the input port

$$\mathbf{V}_1 = \mathbf{V}_s - 5\mathbf{I}_1 \qquad (3)$$

and at the output port

$$\mathbf{V}_2 = -20\mathbf{I}_2 \quad \Rightarrow \quad \mathbf{I}_2 = -\frac{\mathbf{V}_2}{20} \qquad (4)$$

Substituting Eqs. (3) and (4) into Eq. (1) gives

$$\mathbf{V}_s - 5\mathbf{I}_1 = 22\mathbf{I}_1 - \frac{18}{20}\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{V}_s = 27\mathbf{I}_1 - 0.9\mathbf{V}_2 \qquad (5)$$

while substituting Eq. (4) into Eq. (2) yields

$$\mathbf{V}_2 = 18\mathbf{I}_1 - \frac{30}{20}\mathbf{V}_2 \quad \Rightarrow \quad \mathbf{I}_1 = \frac{2.5}{18}\mathbf{V}_2 \qquad (6)$$

Substituting Eq. (6) into Eq. (15), we get

$$\mathbf{V}_s = 27 \times \frac{2.5}{18}\mathbf{V}_2 - 0.9\mathbf{V}_2 = 2.85\mathbf{V}_2$$

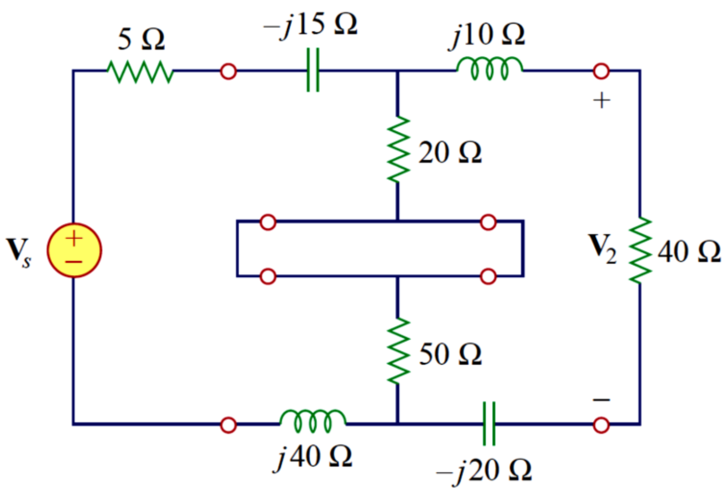
And so,

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{1}{2.85} = 0.3509$$

H.W.12: Find $\mathbf{V}_2 / \mathbf{V}_s$ in the circuit in Fig.

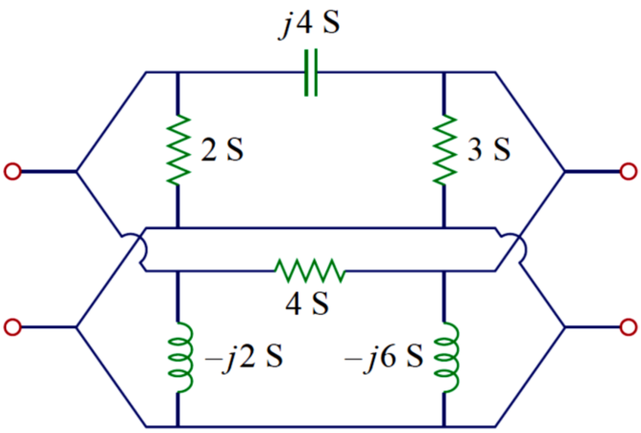
Answer:

$0.6799\underline{/-29.05^\circ}$.



Example 13: Find the *y parameters* of the two-port in Fig.

Solution:



Let us refer to the upper network as N_a and the lower one as N_b. The two networks are connected in parallel. Comparing N_a and N_b with the circuit in Fig. 3.2(a), we obtain

$$\mathbf{y}_{12a} = -j4 = \mathbf{y}_{21a}$$

$$\mathbf{y}_{11a} = 2 + j4$$

$$\mathbf{y}_{22a} = 3 + j4$$

or

$$[\mathbf{y}_a] = \begin{bmatrix} 2 + j4 & -j4 \\ -j4 & 3 + j4 \end{bmatrix} \text{S}$$

and

$$\mathbf{y}_{12b} = -4 = \mathbf{y}_{21b}, \qquad \mathbf{y}_{11b} = 4 - j2, \qquad \mathbf{y}_{22b} = 4 - j6$$

or

$$[\mathbf{y}_b] = \begin{bmatrix} 4 - j2 & -4 \\ -4 & 4 - j6 \end{bmatrix} \text{S}$$

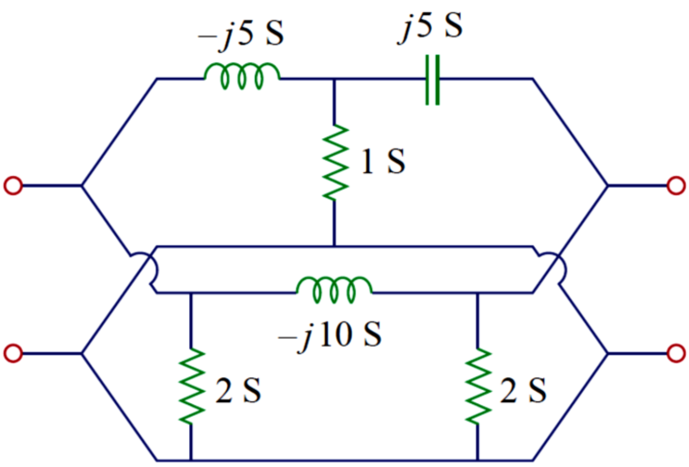
The overall y parameters are

$$[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b] = \begin{bmatrix} 6 + j2 & -4 - j4 \\ -4 - j4 & 7 - j2 \end{bmatrix} \text{S}$$

H.W.13: Obtain the y parameters for the network in Fig.

Answer:

$$\begin{bmatrix} 27 - j15 & -25 + j10 \\ -25 + j10 & 27 - j5 \end{bmatrix} \text{S}$$



Example 14: Find the transmission parameters for the circuit in Fig.

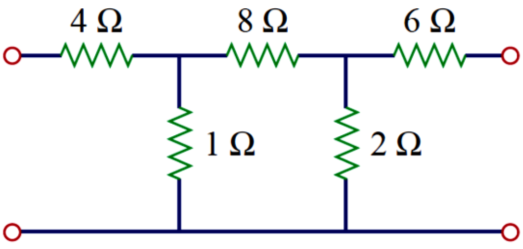


Figure 1

Solution:

We can regard the given circuit in Fig. 1 as a cascade connection of two T networks as shown in Fig. 2(a). We can show that a T network, shown in Fig. 2(b), has the following transmission parameters

$$\mathbf{A} = 1 + \frac{R_1}{R_2}, \qquad \mathbf{B} = R_3 + \frac{R_1(R_2 + R_3)}{R_2}$$

$$\mathbf{C} = \frac{1}{R_2}, \qquad \mathbf{D} = 1 + \frac{R_3}{R_2}$$

Applying this to the cascaded networks N_a and N_b in Fig. 2(a), we get

$$\mathbf{A}_a = 1 + 4 = 5, \qquad \mathbf{B}_a = 8 + 4 \times 9 = 44\ \Omega$$

$$\mathbf{C}_a = 1\ \text{S}, \qquad \mathbf{D}_a = 1 + 8 = 9$$

or in matrix form,

$$[\mathbf{T}_a] = \begin{bmatrix} 5 & 44\ \Omega \\ 1\ \text{S} & 9 \end{bmatrix}$$

and

$$\mathbf{A}_b = 1, \qquad \mathbf{B}_b = 6\ \Omega, \qquad \mathbf{C}_b = 0.5\ \text{S}, \qquad \mathbf{D}_b = 1 + \frac{6}{2} = 4$$

i.e.,

$$[\mathbf{T}_b] = \begin{bmatrix} 1 & 6\ \Omega \\ 0.5\ \text{S} & 4 \end{bmatrix}$$

Thus for total network in Fig. 1,

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] = \begin{bmatrix} 5 & 44 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0.5 & 4 \end{bmatrix}$$

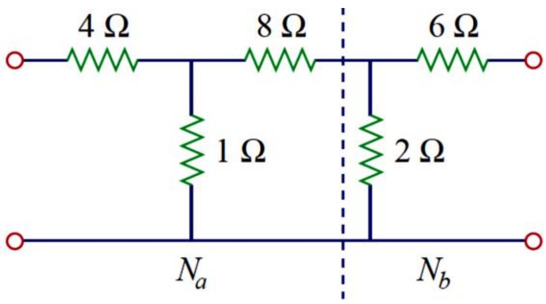
$$= \begin{bmatrix} 5 \times 1 + 44 \times 0.5 & 5 \times 6 + 44 \times 4 \\ 1 \times 1 + 9 \times 0.5 & 1 \times 6 + 9 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 206\ \Omega \\ 5.5\ \text{S} & 42 \end{bmatrix}$$

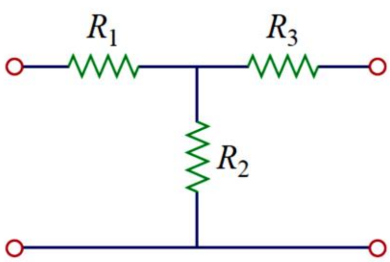
Notice that

$$\Delta_{T_a} = \Delta_{T_b} = \Delta_T = 1$$

showing that the network is reciprocal.



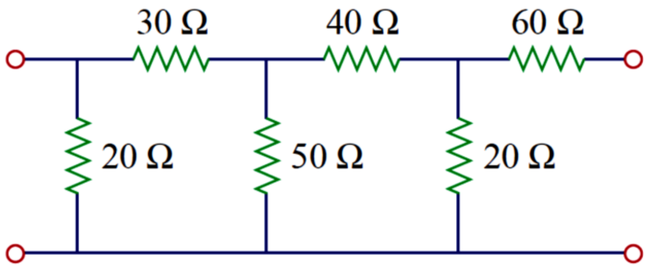
(a)



(b)

Fig. 2 (a) Breaking the circuit in Fig. 1 into two two-ports, (b) a general T two-port.

H.W.14: Obtain the **ABCD** parameter representation of the circuit in Fig.



Answer:

$$[\mathbf{T}] = \begin{bmatrix} 29.25 & 2200\ \Omega \\ 0.425\ \text{S} & 32 \end{bmatrix}$$

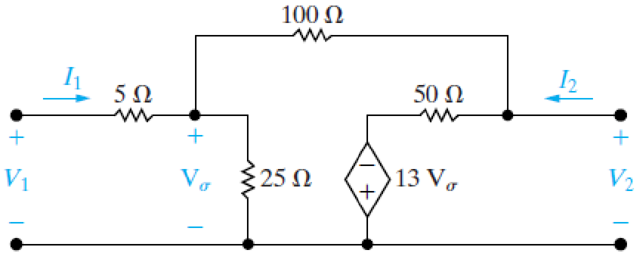
Lecture ()

Two-Port Networks

Problems

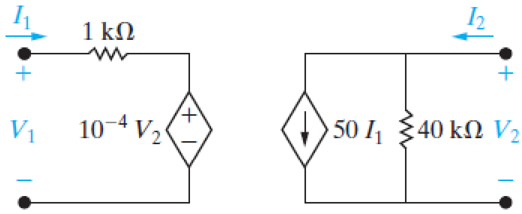
H.W. (1): Find the g parameters for the operational amplifier circuit shown in Fig.

[Answer: g_{11} = 80 mS, g_{12} = -0.2, g_{21} = -5, g_{22} = 25 Ω,]



H.W. (2): Find the transmission parameters for the circuit in Fig.

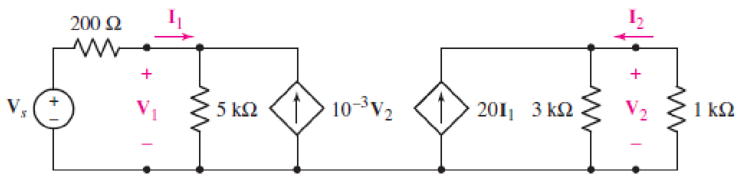
[Answer: A = -4×10^{-4}, B = -20, C = -0.5 µS, D = -0.02]



H.W.(3): Find **y** and $\mathbf{Z}_{out}$ for the terminated two-port shown in Fig.

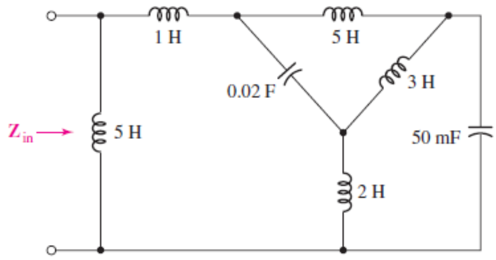
[Answer: 17.6

: $\begin{bmatrix} 2\times10^{-4} & -10^{-3} \\ -4\times10^{-3} & 20.3\times10^{-3} \end{bmatrix}(S)$**]**



H.W.(4): Determine the input impedance Z_{in} of the one-port shown in Fig. if ω is equal to $(a) 50 rad/s$; (b) $1000\ rad/s$.

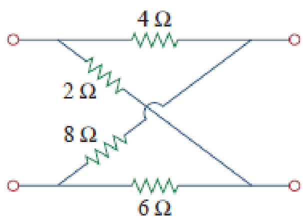
[Answer:]



H.W.(5): Determine the z and y parameters for the circuit in Fig.

[Answer:]

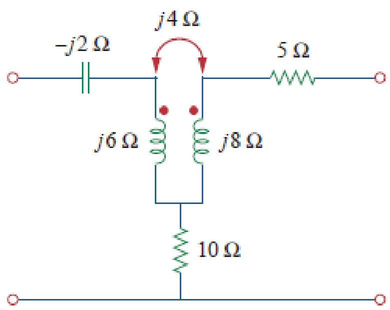
$$[z]=\begin{bmatrix} 4.8 & -0.4 \\ -0.4 & 4.2 \end{bmatrix}\Omega \quad , \quad [y]=\begin{bmatrix} 0.21 & 0.02 \\ 0.02 & 0.24 \end{bmatrix}S$$



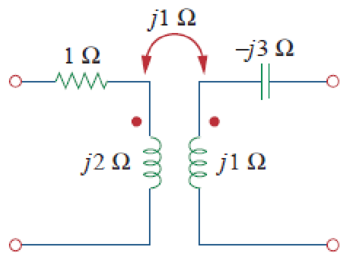
H.W.(6): Obtain the z parameters for the network in Fig.

[Answer:]

$$[z]=\begin{bmatrix} (10+j4) & -(10+j4) \\ -(10+j4) & (15+j8) \end{bmatrix}\Omega$$



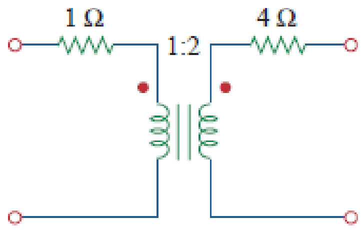
H.W.(7): Obtain the t parameters for the network in Fig.



[Answer:]

$$[t]=\begin{bmatrix} 2 & 2+j5 \\ j & -2+j \end{bmatrix}$$

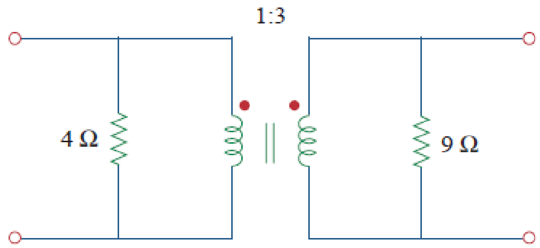
H.W.(8): Determine the z and h parameters for the network in Fig.



[Answer:]

$$[h]=\begin{bmatrix} 2\,\Omega & 0.5 \\ -0.5 & 0 \end{bmatrix}$$

H.W.(9): Determine the z and h parameters of the two-port in Fig.

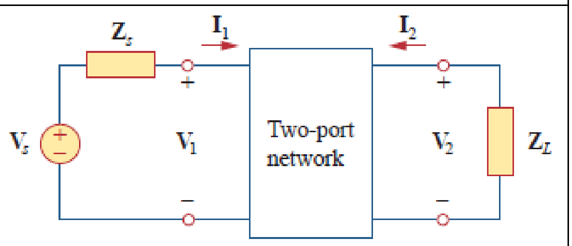


[Answer:]

$$[z]=\begin{bmatrix} 0.8 & 2.4 \\ 2.4 & 7.2 \end{bmatrix}\Omega$$

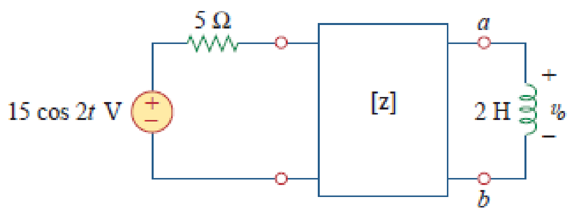
H.W.(10): For the two-port network shown in Fig., show that at the output terminals,

$$Z_{Th}=z_{22}-\frac{z_{12}z_{21}}{z_{11}+Z_s} \quad \text{and} \quad V_{Th}=\frac{z_{21}}{z_{11}+Z_s}v_s$$

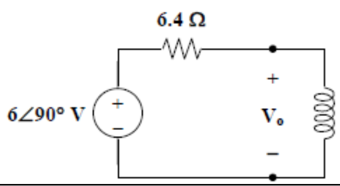


[Answer:]

H.W.(11): For the circuit in Fig., at $\omega=2rad/s$, $z_{11}=10\Omega$, $z_{12}=z_{21}=j6\Omega$, $z_{22}=4\Omega$. Obtain the Thevenin equivalent circuit at terminals a-b and calculate v_o.

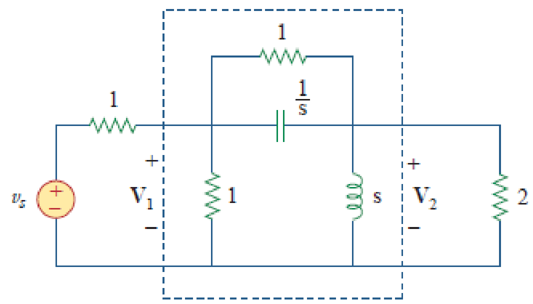


[Answer:]



$$v_o(t)=3.18\cos(2t+148^\circ)\text{ V}$$

H.W.(12): **(a)** Find the y parameters of the two-port in Fig. **(b)** Determine $V_2(s)$ for $v_s=2u(t)$ V.



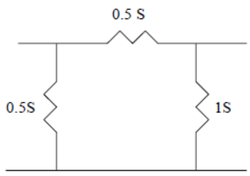
[Answer:]

$$[y]=\begin{bmatrix} s+2 & -(s+1) \\ -(s+1) & \dfrac{s^2+s+1}{s} \end{bmatrix}, \qquad V_2=\frac{0.8(s+1)}{(s^2+1.8s+1.2)}$$

H.W.(13): Draw the two-port network that has the following y parameters:

$$[y] = \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1.5 \end{bmatrix} S$$

[Answer:]



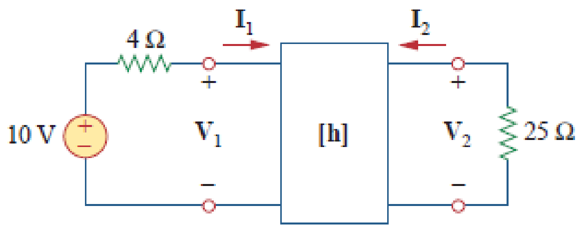
H.W.(14): For the two-port in Fig.,

$$[h] = \begin{bmatrix} 16\Omega & 3 \\ -2 & 0.01\ S \end{bmatrix}$$

Find:

(a) V_2/V_1 (b) I_2/I_1

(c) I_1/V_1 (d) V_2/I_1



[Answer:]

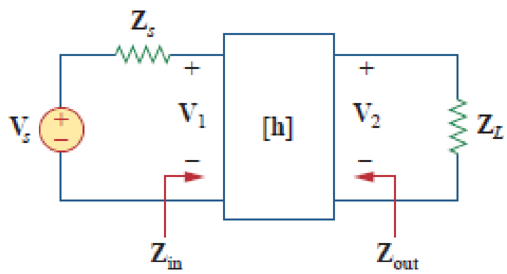
(a) $\dfrac{V_2}{V_1} = \dfrac{40}{136} = \underline{0.2941}$ (c) $\dfrac{I_1}{V_1} = \dfrac{1}{136} = \underline{7.353 \times 10^{-3}\ S}$

(b) $\dfrac{I_2}{I_1} = \underline{-1.6}$ (d) $\dfrac{V_2}{I_1} = \dfrac{40}{1} = \underline{40\ \Omega}$

H.W.(15): The h parameters of the two-port of Fig. are:

$$[h] = \begin{bmatrix} 600\Omega & 0.04 \\ 30 & 2\ mS \end{bmatrix}$$

Given the $Z_s = 2k\Omega$ & $Z_L = 400\Omega$, Find Z_{in} and Z_{out}.

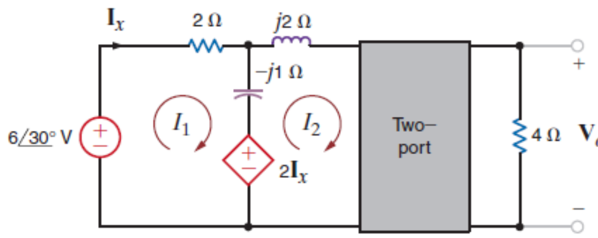


[Answer: $Z_{in} = 333.33\ \Omega$, $Z_{out} = 650\ \Omega$]

H.W.(16): Determine the output voltage V_o in the network in Fig. if the Z parameters for the two- port are

$$Z = \begin{bmatrix} 5 & 4 \\ 4 & 12 \end{bmatrix}$$

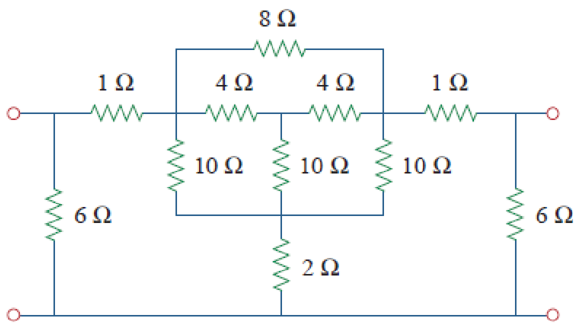
[Answer:]



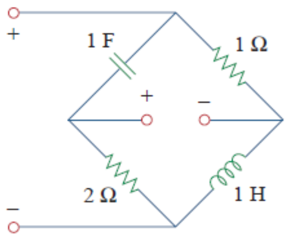
H.W.(17): Obtain the z parameters of the network in Fig.

[Answer:]

$$[z] = \begin{bmatrix} 3.949 & 1.122 \\ 1.122 & 3.849 \end{bmatrix} \Omega$$



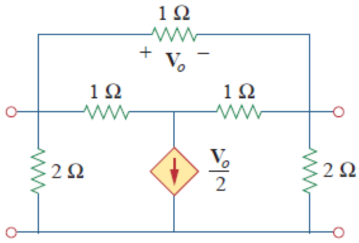
H.W.(18): Find the h parameters of the network in Fig. Take $\omega = 1rad/s$.



[Answer:]

$$[h] = \begin{bmatrix} 0.9488\angle -161.6^\circ & 0.3163\angle 18.42^\circ \\ 0.3163\angle -161.6^\circ & 0.9488\angle -161.6^\circ \end{bmatrix}$$

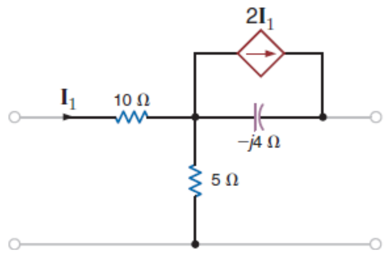
H.W.(19): Find the transmission parameters for the network in Fig.



[Answer:]

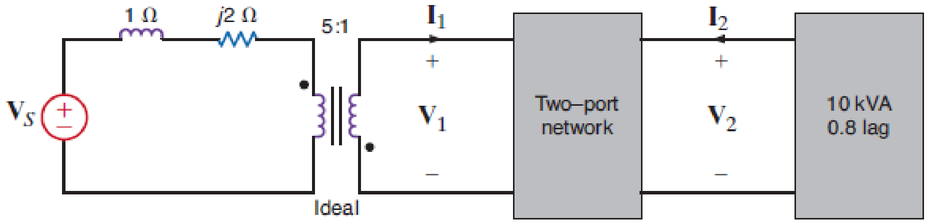
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.4 & -0.8 \\ 1.4 & -1.8 \end{bmatrix}$$

H.W.(20): Find the Z parameters for the two-port network shown in Fig.



[Answer: z_{11} = 15 Ω; z_{12} = 5 Ω; z_{21} = 5 - j8 Ω; z_{22} = 5 - j4 Ω]

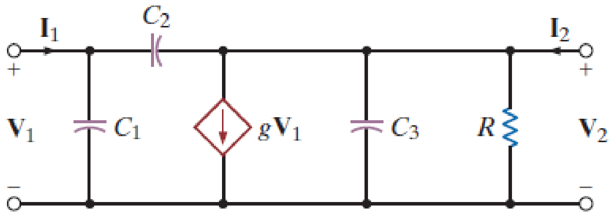
H.W.(21): Find V_s if $V_2 = 220\angle 0^\circ$ V rms in the network shown in Fig.



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 0.333 + j0.333 & -(1.333 + j6) \\ j0.1667 & -(0.333 + j0.333) \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

[Answer: $V_s = 1015.9\angle -137.63^\circ$ V rms.]

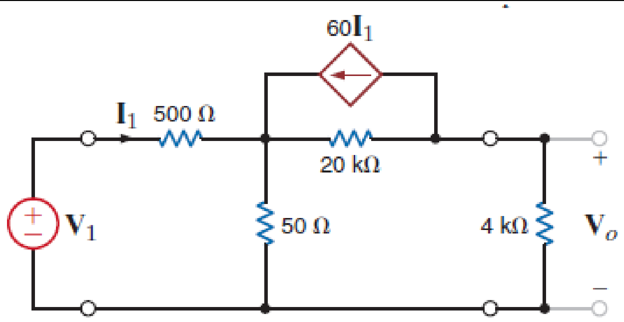
H.W.(22): Determine the admittance parameters for the network shown in Fig.



[Answer:

$$[y] = \begin{bmatrix} j\omega(C_1 + C_2) & -j\omega C_2 \\ g - j\omega C_2 & \frac{1}{R} + j\omega(C_2 + C_3) \end{bmatrix}]$$

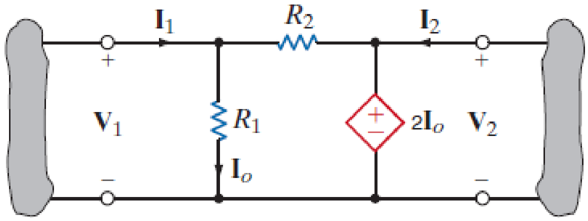
H.W.(23): Determine the z parameters of the network shown in Fig. and determine the voltage gain of the entire circuit with 4 kΩ load.



[Answer:

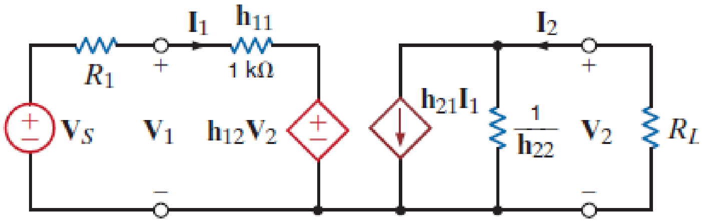
$$[z] = \begin{bmatrix} 550\ \Omega & 50\Omega \\ -1.2\ M\Omega & 20.05\ k\Omega \end{bmatrix}, \frac{V_O}{V_1} = 65 \cdot 5]$$

H.W.(24): Show that the network in Fig. does not have a set of Y parameters unless the source has an internal impedance.



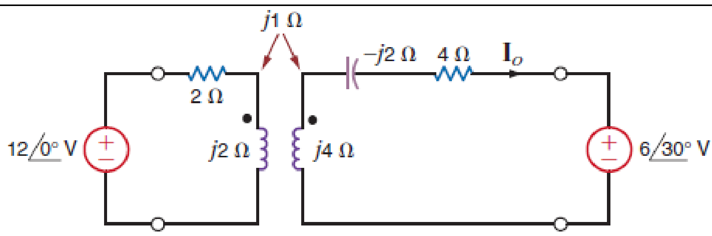
[Answer: $y_{11} = y_{12} = y_{21} = y_{22} = \infty$]

H.W.(25): Consider the hybrid model for a basic transistor. Determine the voltage gain of the entire network, V_2/V_S.



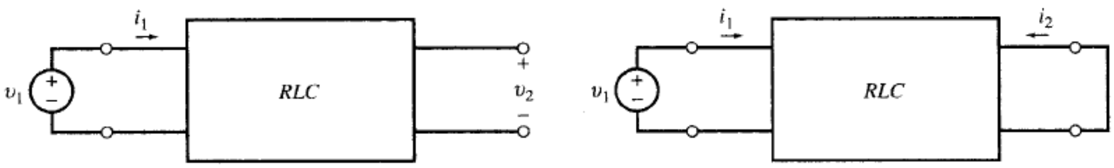
[Answer:
$$\frac{V_2}{V_s} = \frac{h_{21}R_L}{h_{12}h_{21}R_L - (1 + h_{22}R_L)(h_{11} + R_L)}$$ **]**

H.W.(26): Find the transmission parameters for the two-port network and then find I_o using the terminal conditions.



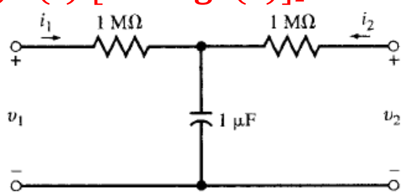
[Answer: :
$$[T] = \begin{bmatrix} 2 - j2 & (12 - j5)\ \Omega \\ -j\ S & 2 - j4 \end{bmatrix}$$ **]**

H.W.(27): A two-port network contains resistors, capacitors, and inductors only. With port 2 open [Fig.(a)], a unit step voltage $v_1 = u(t)$ produces $i_1 = e^{-t}u(t)(\mu A)$ and $v_2 = (1 - e^{-t})u(t)(V)$. With port 2 short- circuited [Fig.(b)], a unit step voltage $v_1 = u(t)$ delivers a current $i_1 = 0.5(1 + e^{-2t})u(t)(\mu A)$. Find i_2 and the T-equivalent network.

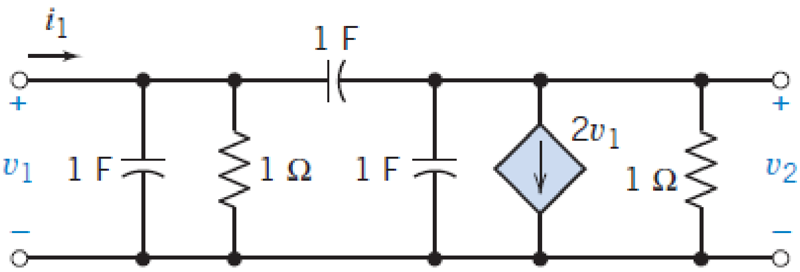


(a) (b)

[Answer: $i_2 = 0.5(-1 + e^{-2t})u(t)$ [see Fig. (c)]]

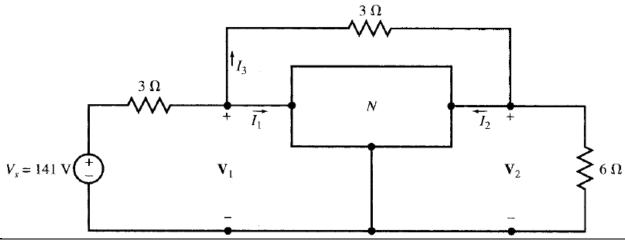


H.W. (28): (a) For the circuit shown in Fig., determine the two-port Y model using impedances in the s-domain. **(b)** Determine the response v2(t) when a current source i_1 =1 u(t) A is connected to the input terminals.



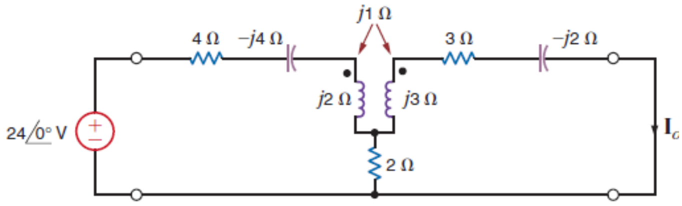
[Answer:]

H.W.(29): The two-port network N in Fig. is specified by Z_{11} =2, Z_{12} = Z_{21} = 1, and Z_{22} = 4. Find I_1, I_2, and I_3.



[Answer: I_1 = 24 A, I_2 = 1.5 A, and I_3 = 6.5 A]

H.W.(30): Find the *Z* parameters for the two-port network in Fig. and then determine I_o for the specified terminal conditions.



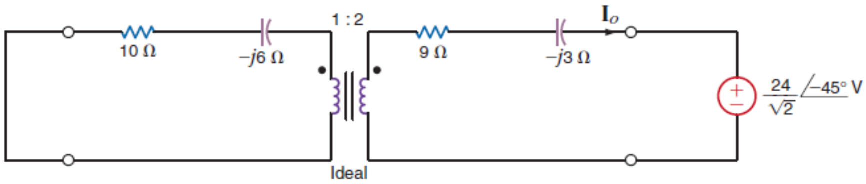
[Answer:]

$Z_{11} = 6 - j2\ \Omega$ $Z_{12} = 2 + j1\ \Omega$

$Z_{21} = 2 + j1\ \Omega$ $Z_{22} = 5 + j1\ \Omega$

$I_o = 1.78 \angle 42^\circ$ A

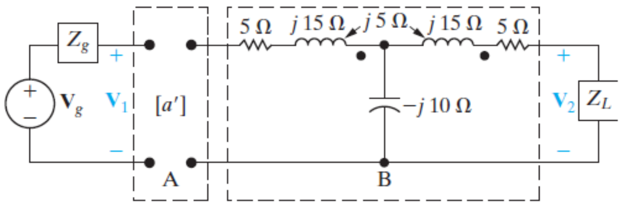
H.W.(31): Find the transmission parameters of the two-port in Fig. and then use the terminal conditions to compute I_o.



[Answer:]

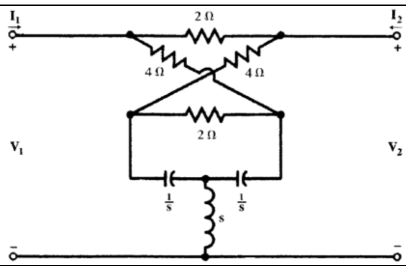
H.W.(32): The networks A and B in the circuit in Fig. are reciprocal and symmetric. For network A, it is known that a =5 and b =24 Ω.

a) Find the transmission parameters of network B. **b)** Find $\mathbf{V_2}$ when $\mathbf{V_g} = 75\angle 0^0$ V, $\mathbf{Z_g} = 1\angle 0^0\ \Omega$, and $\mathbf{Z_L} = 10\angle 0^0\ \Omega$.



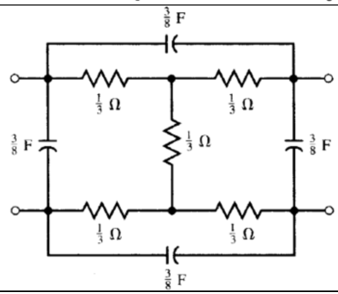
[Answer: (a) $A = -1 + j1$, $B = -10 + j5\ \Omega$, $C = j0 \cdot 2\ S$, $D = -1 + j1$ (b) $V_2 = -2.95 - j2.79$ V]

H.W.(33): Find the Z-parameters in the circuit of Fig.(Use the series connection rule).



[Answer:
$Z_{11} = Z_{22} = s + 3 + (1/s)$;
$Z_{12} = Z_{21} = s + 1$]

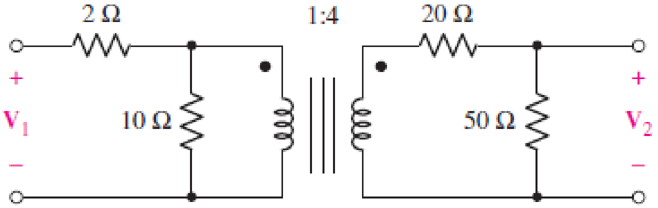
H.W.(34): Find the y-parameters in the circuit of Fig.(Use the parallel connection rule).



[Answer: $y_{11} = y_{22} = 9(s + 2)/16$; $y_{12} = y_{21} = -\ 3(s + 2)/16$]

H.W.(35): By using the rules for interconnecting two-ports in cascade, find **t** for the network of Fig.

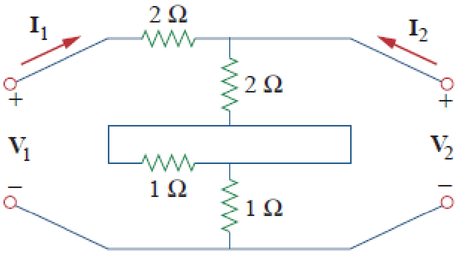
[Answer:]



H.W.(36): What is the y parameter presentation of the circuit in Fig. ?

[Answer:]

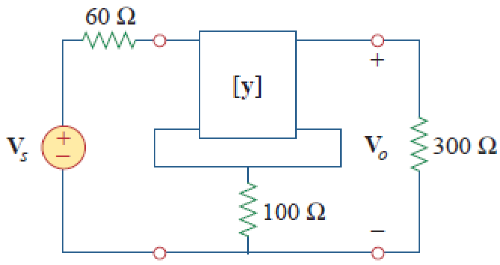
$$[y] = \begin{bmatrix} \frac{1}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} S$$



H.W.(37): In the two-port of Fig., let $y_{12} = y_{21} = 0$, $y_{11} = 2mS$, and $y_{22} = 10mS$. Find V_o/V_s.

[Answer:]

$$\frac{V_o}{V_2} = \underline{0.09375}$$

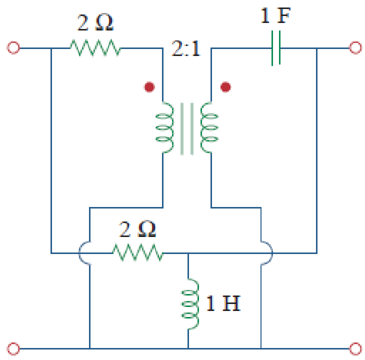


H.W.(38): The circuit in Fig. may be regarded as two two-ports connected in parallel. Obtain the y parameters as functions of s.

[Answer:]

$$[y_a] = \begin{bmatrix} \frac{s}{2(s+2)} & \frac{-s}{s+2} \\ \frac{-s}{s+2} & \frac{2s}{s+2} \end{bmatrix} \qquad [y_b] = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & (s+2)/2s \end{bmatrix}$$

$$[y] = [y_a] + [y_b] = \begin{bmatrix} \frac{s+1}{s+2} & \frac{-(3s+2)}{2(s+2)} \\ \frac{-(3s+2)}{2(s+2)} & \frac{5s^2+4s+4}{2s(s+2)} \end{bmatrix}$$

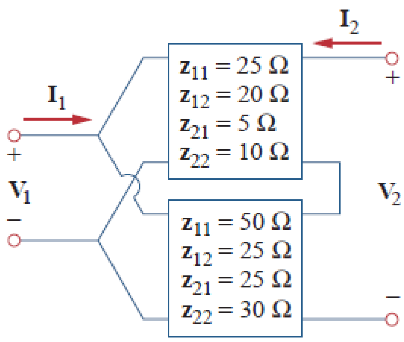


H.W.(39): For the parallel-series connection of the two two-ports in Fig., Find the g parameters.

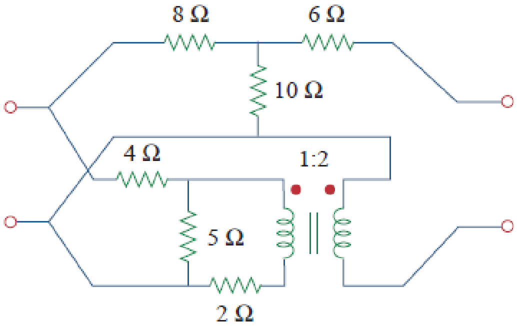
[Answer:]

$$[g_a] = \begin{bmatrix} 0.04 & -0.8 \\ 0.2 & 6 \end{bmatrix}, \qquad [g_b] = \begin{bmatrix} 0.02 & -0.5 \\ 0.5 & 17.5 \end{bmatrix}$$

$$[g] = [g_a] + [g_b] = \begin{bmatrix} 0.06\ S & -1.3 \\ 0.7 & 23.5\ \Omega \end{bmatrix}$$



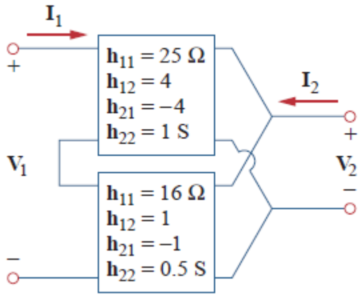
H.W.(40): Determine the z parameters for the network in Fig.



[Answer:]

$$[z] = \begin{bmatrix} 2 & -3.334 \\ 3.334 & 20.22 \end{bmatrix} \Omega$$

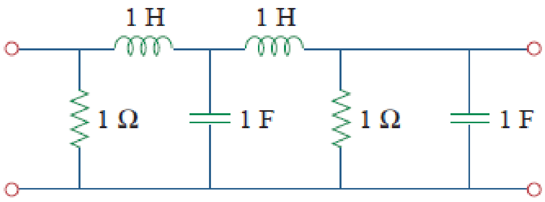
H.W.(41): A series-parallel connection of two two-ports is shown in Fig. Determine the z parameter representation of the network.



[Answer:]

$$[z] = \begin{bmatrix} 57.67 & 3.333 \\ 3.333 & 0.6667 \end{bmatrix} \Omega$$

H.W.(42): Determine the ABCD parameters of the circuit in Fig. as functions of s. (*Hint*: Partition the circuit into sub-circuits and cascade them using)



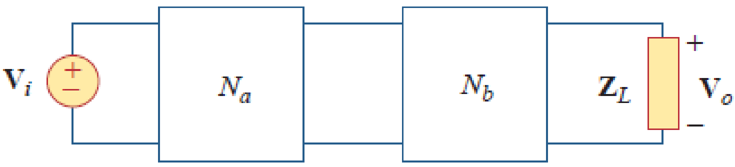
[Answer:]

$$[T] = \begin{bmatrix} s^4 + s^3 + 3s^2 + 2s + 1 & s^3 + 2s \\ s^4 + 2s^3 + 4s^2 + 4s + 2 & s^3 + s^2 + 2s + 1 \end{bmatrix}$$

H.W.(43): For the individual two-ports shown in Fig. where,

$$[z_a] = \begin{Bmatrix} 8 & 6 \\ 4 & 5 \end{Bmatrix} \Omega \; [y_b] = \begin{Bmatrix} 8 & -4 \\ 2 & 10 \end{Bmatrix} S$$

(a) Determine the y parameters ofthe overall two-port. (b) Find the voltage ratio V_o/V_i when $Z_L = 2\Omega$.



[Answer:]

$$[y] = \begin{bmatrix} 0.3015 & -0.1765 \\ 0.0588 & 10.94 \end{bmatrix} , \quad \frac{V_o}{V_i} = -0.0051$$